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Ph.D. Thesis

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Quality-based multi-criteria optimisation approach in dispatch of Services of General Interest

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Abstract

This work contributes to the general research on Decision Support Systems (DSSs) for the dispatchers of the Services of General Interest (SGIs). The SGIs are an important pillar of the social model and of the social market economy of the European Union, centred on guaranteeing the minimum well-being standards for the Europeans. This work focueses on dispatching the SGIs, whose units may be differentiated based on the quality (or speciality) of the service offered. We argue that dispatching the SGIs considering not only the cost of the dispatch but also the SGI quality received by the customers (in a multi-criteria manner), yields more tailored results to the the customers' needs, as opposed to the standard single-criterion dispatch cost minimisation approaches. For this, the generic SGI dispatch optimisation problem with quality-based criteria (SDOQ), aimed to facilitate the dispatcher's decisions, is developed and analysed. Firstly, a theoretical review of the characteristics of the problem is performed and the best suited aggregation method is identified. Secondly, we study statistically the results obtained by solving the SDOQ problem on 6,000 artificial instances with randomly generated parameters — both with and without considering the fairness/equity of the solution. Then, to make the investigations even more realistic, the problem SDOQ is applied to two Case Studies from highly distinct fields, which are based on the previously published papers. The first one is related to the dispatch of electrical energy generating units and the second one to the dispatch of Emergency Medical Services with Emergency Departments. In all of the numerical analyses the performance and dispatch results obtained by solving the proposed approach is compared with the ones obtained by solving the standard single-criterion cost minimisation problems. The comparison is done over six specifically crafted performance measures. The tests have shown that the dispatch results obtained by solving the SDOQ problem have always been more tailored to customers' needs than the ones obtained by solving the standard problems. This however, has been linked to significantly increasing the optimiser's solution time, especially when the fairness of the solution was considered. Nevertheless, despite the increase, the SDOQ has still been solved in acceptable time. To sum up, the proposed SDOQ problem can significantly improve the dispatch results of various SGIs by making the dispatch more tailored to customers' needs.

Keywords: *Services of General Interest, multi-criteria optimisation, dispatch, Decision Support Systems*

Streszczenie

Praca ta mieści się w nurcie badań nad Systemami Wspomagania Decyzji dla dyspozytorów Usług Świadczonych w Interesie Ogólnym (USIO). USIO stanowią ważny filar modelu społecznego i społecznej gospodarki rynkowej Unii Europejskiej, zapewniając Europejczykom minimalne standardy bytowe. W tej pracy badano dyspozycję USIO, których jednostki można rozróżniać między sobą ze względu na oferowaną przez nie jakość (lub specjalistykę) klientom. Przyjęto tezę, że dyspozycja USIO, uwzględniająca nie tylko koszty dyspozycji, lecz także jakość USIO otrzymywaną przez klientów (w sposób wielokryterialny), daje wyniki lepiej dopasowane do ich potrzeb niż w przypadku klasycznej jednokryterialnej minimalizacji kosztów dyspozycji. W tym celu zaproponowano i przeanalizowano generyczny problem optymalizacji dyspozycji USIO z uwzględnieniem kryteriów jakościowych (SDOQ). W pierwszej kolejności dokonano teoretycznej analizy problemu i zidentyfikowano najlepiej dopasowaną metodę agregacji. Następnie przeanalizowano statystycznie wyniki uzyskane przez rozwiązanie problemu SDOQ na 6 000 instancjach z losowo dobranymi parametrami optymalizacji — także uwzględniając sprawiedliwość (ang. *fairness*) rozwiązania. W kolejnym kroku zaaplikowano problem SDOQ do dwóch Studiów Przypadku opartych na wcześniej opublikowanych artykułach. Pierwsze Studium dotyczyło dyspozycji jednostek wytwórczych energii elektrycznej, drugie natomiast dyspozycji Zespołów Ratownictwa Medycznego oraz Szpitalnych Oddziałów Ratunkowych. We wszystkich analizach porównywano wyniki dyspozycji uzyskiwane przez proponowane podejście oparte na SDOQ z wynikami otrzymywanymi przez klasyczny jednokryterialny problem minimalizacji kosztów dyspozycji. Porównania dokonano w oparciu o sześć specjalnie opracowanych miar wydajności. Testy wykazały, że wyniki dyspozycji uzyskiwane w wyniku rozwiązania problemu SDOQ były lepiej dopasowane do potrzeb klientów, niż te uzyskiwane w wyniku rozwiązania problemu klasycznego. Wiązało się to jednak z istotnym statystycznie wydłużeniem czasu obliczeń, zwłaszcza w przypadku sformułowania uwzględniającego sprawiedliwość rozwiązania. Jednakże, mimo że zwiększony, czas ten nadal pozostawał akceptowalny. Podsumowując, zaproponowany problem SDOQ może znacząco poprawić wyniki dyspozycji różnych USIO, czyniąc je lepiej dostosowanymi do potrzeb klientów.

Słowa kluczowe: *Usługi Świadczonych w Interesie Ogólnym, optymalizacja wielokryterialna, dyspozycja, Systemy Wspomagania Decyzji*

Contents

Chapter 1

Introduction

1.1 Background

1.1.1 Services of General Interest (SGIs)

Services of General Interest (SGI) are identified by the institutions of the European Union (EU) as a supporting pillar of the European social model and of the European social market economy. This term includes services such as: electricity or water supply, healthcare, social services, telecommunications and broadcasting, sewage disposal, public transport, postal services, housing or others [1]. Simply put, any service that is essential for the lives of the majority of EU citizens will fall into this category [2].

The European Union legislation sets grounds for the term *SGIs* and for the concepts, which describe it further. [1] These concepts are presented by the European Commission in the Quality Framework for Services of General Interest in Europe [6]. The Framework [6] defines SGIs as *services that public authorities of the Member States classify as being of general interest and, therefore, subject to specific public service obligations (PSO). The term covers both economic activities and non-economic services.* The same document then defines a sub-class of the same as *Services of General Economic Interest (SGEI)*. This term is defined as ***economic*** *activities which deliver outcomes in the overall public good that would not be supplied (or would be supplied under different conditions in terms of quality, safety, affordability, equal treatment or universal access) by the market without public intervention. The PSO is imposed on the provider by way of an entrustment and on the basis of a general interest criterion which ensures that the service is provided under conditions allowing it to fulfil its mission.* Then, the Framework also defines another sub-class of SGIs, being the Social Services of General Interest (SSGIs). Their definition is as follows: *services including social security schemes covering the main risks of*

[1] Legislative grounds are given in protocol number 26 to the Treaty on the Functioning of the European Union (TFEU). This protocol concerns SGIs but does not define the concepts [3]. The grounds of the SGIs are also described in Articles 14 and 106 of the TFEU and in Art. 36 of the Charter of Fundamental Human Rights [4, 5].

life and a range of other essential services provided directly to the person that play a preventive and socially cohesive/inclusive role. The term SSGI covers both economic and non-economic activities. The relations given in the definitions above are depicted in the Venn diagram in Fig. 1.1.

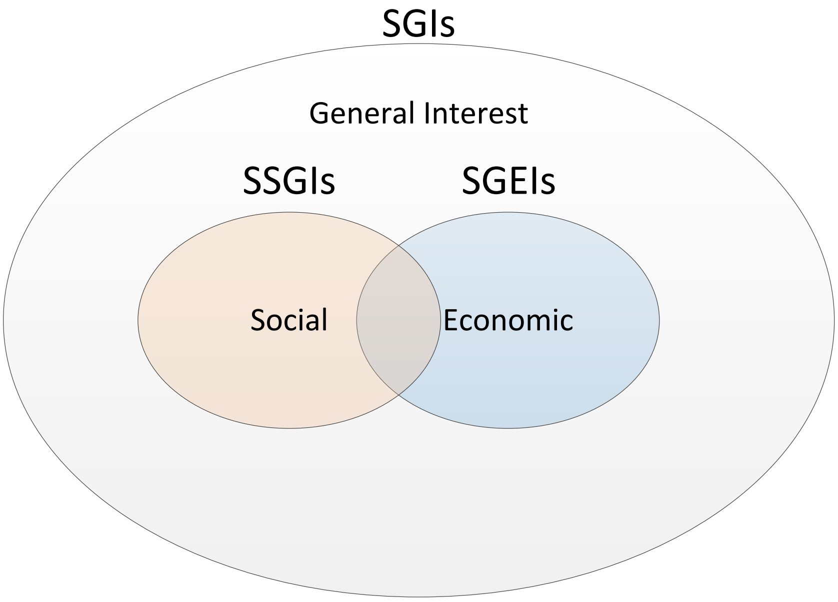


Figure 1.1: Relations between SGIs, SGEIs ans SSGIs (own elaboration basing on [3])

One may notice that the definitions of SGIs is close to the one of *public services*. This is true and already recognised by the European Commission in the same Framework. The Commission states that in the EU legislation the term *public services* is used in a rather ambiguous way, and mostly in applications related to the transportation services. The ambiguity arises as this term is used for services designed for the public interest, as well as to the activity of public entities. Therefore, the Commission proposes to use the term SGIs, and so it is followed in this work.

According to the SGI Europe organisation, *SGIs guarantee minimum well-being standards for everyone, making lives better and less dependent on external factors and circumstances*. The organisation outlines that they played a vital role in the COVID-19 pandemic times, when the SGIs remained operational, whereas all non-essential economic activities were halted due to lockdowns. The importance of the SGIs is especially visible as it is estimated that around 60 million workers are employed in the European Union in this sector, and they contribute to around 26% of the total EU's Gross Domestic Product (GDP) [7].

Having said the above, one can conclude that the SGIs are an important pillar of the lives of the EU citizens and the EU's economy. In this work we focus solely on services with units dispatched or allocated as a result of specific (physical or virtual) dispatchers' decisions. Therefore, we would normally exclude SSGIs as they mostly concern services such as: social security schemes in various risks in life, e.g., retirement, ageing, occupational accidents, unemployment or disability [6]. Therefore, in the remainder of this work whenever we refer to SGIs we mean the dispatchable services, excluding the SSGIs.

In this place, we harmonise the term *dispatch* or *dispatching* as the action of the same. The Cambridge Dictionary of English defines the verb *to dispatch* as *to send something, especially goods or a message, somewhere for a particular purpose*, and the noun *dispatch* as the action of doing the same [8]. In the operational research literature this term is used for problems of either allocating of a given pool of resources to some tasks, jobs or services or for problems of calculating the setpoints the same, such that the system meets its functional requirements, e.g., covers the demand for the required resources. These problems may be solved for a single time period, as in the vehicle dispatch problems presented in [9, 10], or for a time horizon constituted of multiple periods, as in the dispatch of electrical energy generation plants given in [11, 12]. In the light of the above, in this work we refer to *dispatch*, whenever a given SGI is allocated by the dispatcher to meet system's requirements, regardless of the time horizon of such an action.

1.1.2 Operating the SGIs

Operating the SGIs is a nontrivial and complex job. They are essential for smooth functioning of the society and should be accessible to all citizens in a fair manner. A given EU Member State should accomplish multiple tasks in defining the operations of such services. Firstly, the state should define what constitutes the *general interest* and what exact services fall under this category. Some of the services have been harmonised at the European level (e.g., telecommunications, postal and energy sectors) and some are left to the discretion of the states, with some bounds imposed by the European legislation [3]. Once done, the State should then architect the operations of the same and construct the public service obligations bounding their providers. After completing the architectural and legal steps, the State should identify the optimal location, types and the pool of units of the service considered. In case some of them are delivered via dispatchable units, dispatch policies with further key performance indicators of the same should be established.

In the majority of tasks mentioned above, decisions must be made by the respective Decision Makers (DMs). Those decision processes are often complex and can be facilitated by using various computer-based Decision Support Systems (DSSs) with appropriate mathematical modelling implemented. We give more information on those in the next Section of this work.

Dispatching of the units itself is also a multi-stage task. Firstly, special legal bounds for it should be created. Once done, dispatch policies should be established. They can only work, when appropriate ways and tools of continuous monitoring of the system are defined, with the assessment of dispatch decisions. Only then, the dispatcher may take its role and allocate the required units to meet the functional requirements. Thus, although this action founds the core of dispatch actions, it is only a step in the whole dispatch organisation. Despite the fact that the scope of SGI operations is broad, this work treats only the tasks of decisions on dispatching units of SGIs. Deliberately only those are discussed further in this Section.

To dispatch the units in the most appropriate and efficient way, the dispatcher must have some deep domain knowledge and expertise on the SGI application, together with as precise visibility of the current operational state of the system as possible. One must remember that those services should always work reflecting citizens' preferences, assuring that the entire system operates correctly and that the service itself is available to all citizens. What is more, while identifying the operations, general interest must also be practically considered by the correct entity. Therefore, Decision Support Systems (DSSs), problems and models that could possibly ease this activity have been of research interest for some long time already. In particular, DSS systems have been widely used for calculating allocation and dispatch of services such as mentioned in the points below:

- electrical energy supply systems [13];
- emergency crews — e.g., Emergency Medical Service, Police, Fire Brigade [14];
- gas supply systems [15];
- waste treatment systems [16];
- telecommunication networks [17];
- public transportation systems [18];
- water supply systems [19].

The act of dispatching in the cited examples of DSSs are based on mathematical optimisation models. This method is particularly interesting to facilitate the dispatch of SGIs, as it allows to find an optimal setting of feasible controls by considering the physical characteristics of the complete SGI system in the form of constraints. Taking the example of an electrical energy supply system, selected constraints may include the operational limits of generating units, capacities of transmission lines or voltage levels in nodes, as approached in our paper [20]. Similarly, those can also be identified in other applications. For example, in the emergency crew dispatching problem the dispatcher may consider the availability of crews, equipment carried on board of ambulances, availability of hospital beds or levels of competence towards treating a particular medical emergency.

Apart from the possibility of modelling the various sets of physically feasible controls, mathematical optimisation gives the ability to dispatch SGI units in a way that enhances the operating point of the system under consideration through appropriate manipulations in the objective function. This approach was applied by us for instance in [21], where we proposed to re-dispatch the electric SGI basing on reactive power generation to enhance the level of the steady-state voltage stability of the power system considered. Technical issues of power transmission were also addressed through adjustments of the objective function in our paper [22].

1.1.3 Decision Support Systems (DSSs)

We have already stated that the operation of SGIs is nontrivial and is constituted of multiple decision stages. We eluded that these actions can be facilitated by the use of appropriate DSS. In this Section we give some background to the DSS theory.

Decision Support Systems is a research area that falls within the discipline of information systems that focuses on supporting and improving the managerial decision making. It mainly looks at developing and implementing IT-based systems to support the (managerial) decision process [23]. The research history related to it dates back to the mid-1960s and the term was firstly used in 1971 by Scott Morton and Gorry [24, 25]. Depending on how they operate, the DSSs can be either *active*, *passive* or *cooperative*. The active ones support the decision process and generate suggestions of decisions to a problem, whereas the passive ones do not produce any suggestions. The cooperative DSSs work in interaction with the user. They present the decision suggestions to the user for review and refinement, then the systems further improve them prior to the final validation by the user [26].

According to Power [25], DSSs can also be categorised based on how they are built into five groups. The first kind are the *model-driven* DSSs. They emphasise the access to and manipulation of financial, optimisation and/or simulation models. Digital Twins used for the what-if situation can be named examples of those [27]. The second category is *data-driven* DSSs. Such systems emphasise access to and manipulation of the time series of the internal and sometimes external and real-time data. Currently, the trend in those systems is put on the *Business Intelligence* dashboards [28]. Power also defines *communication-driven* DSSs, which use network and communication technologies to facilitate decision-relevant communication and communication (e.g., groupware or tele conferencing systems). The fourth category is *document-driven* DSSs, which use computer storage and processing technologies to provide document retrieval and analysis. The primary example of it is the search engine of documents. To make such a document-driven DSS work effectively, Kacprzyk and Zadrożny propose a method to categorise the available documents using fuzzy linguistic summaries [29]. Finally, the last category of DSSs identified by Power is *knowledge-driven* DSSs. These are systems with a particular problem-solving engine, that consists of domain-specific expertise and knowledge, together with having problem-solving skills appropriate to this domain. They integrate Artificial Intelligence (AI)-based systems, expert systems, data mining and communication systems for the decision support task [26]. An example of an AI-based DSS can be found in [30], where it was applied to the trading of the stocks.

Even though the five categories have been given by Power, some hybrid DSSs may also exist. An example of such hybrids is developed in [31], where the Authors combined data-driven and knowledge-driven DSSs into one tool or in [32], where model-diven and knowledge-driven DSSs were combined.

As per the architecture of those systems, Smutnicki [33] states that DSSs should be equipped at least with a human-machine interface, database management tools, model checker, multipurpose solving engine with logic that focuses on the given application, manager of the solving process and a module evaluating the obtained results. Such definition falls perfectly within the basic architectural components of DSSs, identified by Holsapple [34]. The Author defines that the DSS are built of four main blocks, namely: *language system* — consists of all messages a DSS can accept; *presentation system* — consists of all messages a DSS can emit; *knowledge systems* — consists of all knowledge a DSS has stored and retained; p*roblem-processing system* — engine that processes and solves the decision problems.

A good example of a DSS is given by Michałowski et al. [35], where the Authors developed a system that helped in categorising the paediatric patients, who presented to an Emergency Department with abdominal pain. System was built in the client-server architecture with handheld Palm devices serving as clients. Detailed description of the architecture and the decision algorithms may be found in the reference.

1.1.4 Notion of quality

According to the definitions of SGIs given in Sec. 1.1.1, the public interventions should ensure the best *quality* of the SGIs received. The document [6] precises that the European Commission encourages the provisioning of SGIs to the European in a high-quality manner and the document [3] states, that democratic choices of SGI quality by the Europeans should be guaranteed. Having this said, let us now dive a little deeper into the notion quality in this Section. In the following Sections we elaborate also how quality can be integrated in the DSSs for the SGI dispatch optimisation problem.

According to the Cambridge Dictionary of English available online, *quality (noun)* is defined as: *a characteristic or feature of something that makes it different from other things; how good or bad something is; a high standard; a good feature of a person's character* [36]. In other words, the term describes a differentiator or high standard of something — in the case of this work of an SGI service more precisely.

Generally, the notion of quality is found in many fields of engineering. In power systems engineering, this term is widely used when referring to current, voltage, power quality, as well as Quality of Supply and Quality of Consumption. Some of these are linked to not deviating the actual current/power/voltage values from their ideal operational point. Some of the others, namely for Supply and Consumption, also include the non-technical aspects of the interaction between supply and demand sides [37]. Examples of adding a quality component into power system dispatch and planning optimisation are given in [21, 38].

Similarly, quality is considered in telecommunications under the term *Quality of Service (QoS)*. Denda et al. [39] define this term in a general form as some *abstract user requirements on the data delivery*. This definition is in line with the definition proposed by the International Telecommunication Union (ITU), which says that the QoS is: *The collective effect of service performances which determine the degree of satisfaction of a user of the service* [40]. Wide adoption of the QoS notion led to building its support mechanism into some network protocols. Some examples of those applicable to the Internet of Things (IoT) may be found in [41]. The introduction of generally understood QoS into telecommunication optimisation problems was also a subject of broad research, specifically including the notion of fairness. This was shown in the survey conducted by Ogryczak et al. [17]. What is more, from another survey which was conducted by Józefowska et al. [42] it becomes apparent that QoS is of importance while optimising for energy consumption of the various Information and Communication Technologies (ICT) systems. Apart from these notable reviews, interesting pieces of research in this field may be found in [43, 44], where Authors optimised for the QoS indicators in a multi-criteria manner.

Apart from the specific quality applications above, we can also find research related to *quality engineering*. This is a scientific discipline which studies aspects related to obtaining and keeping an economically reasonable quality of products and processes [45]. Yet, due to the fact that it is a separate discipline itself, rather loosely linked to the subject of this work, we do not cite any specific references. An interested reader is invited to consult the work by Sałaciński for more information [45].

1.1.5 Multi-criteria dispatch of SGIs

As shown in Sec. 1.1.2 dispatching of SGIs is a difficult and complex job, that can be eased by using model-driven decision support tools basing on mathematical optimisation techniques. In this Section we aim to showcase that considering quality-based criteria in addition to the regular cost-based ones (or time-based) can be integrated well into the dispatch of multiple various SGI classes. What is more, we aim to outline that in each of those classes such consideration may add value to the dispatch customers or more generally participants. Firstly, we elaborate further how quality-based criteria integrate in the dispatch of electrical energy generating units and of Emergency Medical Services/Departments. Secondly, we present a non-extensive list of possible further applications.

With respect to the electrical energy markets, the classic approach is to find a dispatch that minimises the total cost of generation or to maximise the social welfare over the system's technical constraints [46, 47]. However, this approach treats all market participants alike, treating as if all of them wanted to pay as little as possible for the energy delivered. This neglects the fact that there might exist conscious participants willing to receive energy that is of better quality, under a possibly higher cost. Some conscious participants may want to receive energy from more ecological sources or from more socially-responsible generating companies. By dispatching of units considering this phenomenon, one may better reflect participants' preferences on the service received and better integrate their choices. In that sense, dispatching of generating units taking into consideration the quality of energy, together with its cost as criteria could add some significant value for the market participants, instead of treating all of them alike [13].

Similarly, in Emergency Medical Services there is a trend to dispatch the unit located closest to the call, and then to take the patient to the nearest hospital. Yet, some other strategies might also exist such as maximising the total ambulance coverage or preparedness of the system [48, 14, 10]. Nevertheless, these strategies tend to treat all patients the same, regardless of the emergency they are having. Not considering patient's clinical condition in the dispatching process may result in sending a wrong unit to the scene and in transporting the patient to a wrongly assigned hospital. This might further result in the need of re-transferring the patient to a different hospital, considerably prolonging the time-to-treatment. There exist some acute conditions which need to be treated in a specific hospital, within a clearly specified time window from symptom onset for the treatment to be effective. Some examples of those conditions are — aortic dissection, ST-elevation myocardial infarction (STEMI) or massive pulmonary embolism. Prolonging the time-to-treatment may cause the treatment to be not effective [49, 50, 51]. Dispatching of hospital units considering both time-to-arrival and their speciality towards treating a given emergency, which can be otherwise called *quality*, as criteria could possibly reduce the risk described. Similarly, dispatching of ambulance crews considering both time-to-arrival and speciality (quality) of the team (e.g., by having a doctor in) and the equipment onboard (e.g., with electrocardiogram with teletransmission capabilities) can potentially help patients in the acute state.

So far, we have drawn a picture of possible utility of quality-based criteria in the dispatching of Emergency Medical Services and of electrical energy generating units, to produce more results which are more tailored to customers' needs. These applications are studied in detail in Chapters 5 and 6.

However, the field of possible applications of the quality-based multi-criteria problems in the dispatching of SGIs does not stop there. One can also envisage using them in the dispatching of other areas, such as:

- **Heat generating units** in multi-unit and multi-technology district heating networks. In this case, units can be differentiated basing on the level of ecology of the technology applied. There might exist conscious heat consumers who will to pay more just to receive some more ecological heat. Similarly to dispatching of electrical energy generation units, one would consider the ecology of technology as a quality criterion together with the cost of sale. An example of a possible candidate heating network is described in [52].

- **Telecommunication networks.** In those applications it is required to allocate available network resources to a data transfer, such as available bandwidth, to provide them to all communication services and to all pairs of senders-receivers in an optimal way [17]. The notion of the Quality of Service as described in Sec. 1.1.4, and its quantification, could be integrated as a quality-based criterion in the resource allocation optimisation to improve user's sentiment of the communication resource that was allocated (or dispatched) to them. Such a measure focusing on user experience (in a global formulation) for the LTE systems was proposed by Yaacoub and Dawy [53]. Also, the notion of QoS in resource allocation optimisation for the beyond 5G networks was studied by Cao et al. [54].

- **Police, Fire Brigade** and other emergency services. Normally, emergency services such as the Police or Fire Brigade can be distinguished basing on the speciality of units. Some examples of Police units could be: neighbourhood patrol units, traffic patrol, criminal Police, canine, or SWAT (Special Weapons and Tactics) [55, 56, 57]. Similarly, one can think of Fire Brigade units, where different types of fire trucks and different specialised teams (e.g., Hazardous Materials Units or Height Rescue Teams) [58]. As in the EMS dispatching problem, one would consider the time to respond to the call of a given unit as one criterion, and its speciality towards handling a given type of emergency as the quality criterion.

- **Postal Courier services.** Currently, post office couriers operate on various types of vehicles — conventional or electric lorries, city scooters or bicycles. One may envisage the existence of conscious customers, for whom it makes a difference whether their delivery is transported in an eco-friendly vehicle. In that sense, couriers can be dispatched basing on a given quality criterion of eco-friendliness (quality) of their vehicle, as well as on closeness/logistical criterion to the package delivery source or destination (depending on contractual agreements). The use of different types of vehicles in postal services was studied in [59].

- **Taxi services.** Taxi services are part of the city's public transportation system [60]. As such, they enhance the lives of Europeans and can be considered as an SGI. Taxis can be dispatched in a way that the waiting time or the ride cost is minimised and that a given type of taxi vehicle is dispatched (quality criterion). Such a strategy is already in use in applications like Uber [61] or Free Now [62], where the customer can choose whether to order a more or less luxurious car or if to order an eco-friendly vehicle. Depending on the type of vehicle chosen, both the ride costs (which are known upfront) and the waiting times differ.

The list above of possible applications is given for reference only and is non-exhaustive. One can think of other possible applications, provided that they are constituted of dispatchable SGI units, which can be differentiated basing on the quality of the service offered.

1.2 Organisation of the thesis

In this Section, we give some guidance into how this thesis is organised. In Chapter 1 we introduce the research topic, discuss the importance of SGIs, formulate the main research question and formulate the intermediate research objectives. In the same Chapter, we introduce the generic SGI dispatch optimisation problem with quality-based criteria from a theoretical perspective.

After this, in Chapter 2 we review and discuss some helpful literature related to the problem statement and discuss the main concepts related to the multi-criteria optimisation. Those concepts are deemed important for further understanding of the work.

Once the literature is reviewed, with some inspirations gathered, in Chapter 3 we formulate the *generic SGI dispatch optimisation problem with the quality-based criteria*. In the same Chapter multi-criteria solution strategies are discussed with respect to the problem formulated. The conclusions are drawn on the most suitable solution strategy in our opinion.

In Chapter 4, we present numerical experiments designed to verify the main research thesis statistically. For this, a variation of the generic problem is crafted and solved over three Cases, each consisting of 2,000 instances. In total, 6,000 experiments are performed. In each of them, the results obtained by solving the proposed problem are compared with the single-criterion minimal cost approach statistically by looking at five performance indices.

The proposed problem is then put into action to dispatch the electrical energy generating units in a market environment in Case Study I presented in Chapter 5. This is considering the conclusions drawn in previous chapters. Then, Case Study II is presented in Chapter 6, where we investigate the use of the same to dispatch Emergency Medical Services and Emergency Departments.

The study is summarised by conclusions and a discussion of the results presented in Chapter. 7. In this Chapter we show the primary research outcomes, verify the research thesis and objectives and discuss the limitations of the study. The Chapter is finished by presenting the possible directions for further research in the field.

1.3 Research objectives

Having shown the possible applications of multi-criteria approaches to operating the dispatchable SGIs with quality-based criteria, one would aim to study them in greater detail.

Therefore, in this thesis, we formulate the following research thesis:

> ***Adding quality-based criteria to the generic SGI dispatch optimisation problem adds value to the dispatching, allowing to yield more tailored results to the customers' needs.***

To verify the above statement, in this work, we formulate the below intermediate research objectives:

Objective 1: Formulation of the generic multi-criteria SGI dispatch optimisation problem with quality-based criteria.

Objective 2: Applicability analysis of selected aggregation methods to solve the proposed generic multi-criteria SGI dispatch optimisation problem with quality-based criteria.

Objective 3: Comparison of the dispatch results obtained by solving the proposed multi-criteria optimisation problem with quality-based criteria in acceptable time with the standard single-criterion cost minimisation approaches.

Objective 4: Case Study I: Applicability analysis of the proposed multi-criteria problem (with quality-based criteria) to the dispatch optimisation of the electric energy generating units, which participate in the energy market.

Objective 5: Case study II: Applicability analysis of the proposed multi-criteria problem with quality-based criteria to the dispatch optimisation of the Emergency Medical Services and Emergency Departments.

The formulated generic multi-criteria SGI dispatch optimisation problem with quality-based criteria is put into applications in the two Case Studies. The first application studied is the dispatching of electrical energy generating units at the day-ahead energy market, where dispatching happens considering both cost and energy quality criteria. This part is mostly inspired by our paper [13]. This is given in Chapter 5.

The second application presented in this work is the dispatching of Emergency Medical Service crews to acute-state patients and the patients (onboard the ambulances) to applicable hospitals. Dispatching happens both considering the time-to-arrival and speciality towards treating a given acute medical condition — both of the ambulance crews and the destination hospitals. This part is mainly inspired by our paper [63]. This is given in Chapter 6.

1.4 Introducing the generic SGI dispatch optimisation problem with quality-based criteria

In this Section we briefly introduce the generic SGI dispatch optimisation problem with quality-based criteria, which forms the Research Objective I of this thesis and is shown in Chapter 3. Its goal is to provide the SGI dispatcher with a tool to support a step of their decision making process — the dispatch of available SGI resources to the customers, reflecting participants' preferences. The generic form, however, is not sufficient to solve all dispatch-related issues in many of the realistic applications. In this Section, the generic problem is described in natural language, from functional and design requirements perspective. Its detailed mathematical formulation however, is given further in Chapter 3, after taking inspirations from the literature.

We have already shown that there exist numerous various dispatchable SGIs, which can be differentiated by the quality of service they offer to the dispatch participants. Basing on this differentiation, we may consider the quality as criteria additional to the cost/time ones in the dispatch process. We have also shown, that the SGIs should be accessible to all European citizens and should operate guaranteeing democratic choices of dispatch participants [3] (or in other words, considering their personal preferences). At the same time, the dispatch must consider the current operational state of the considered SGI system — allowing to only make technically feasible decisions.

Having recapitulated the operational principles, we may design the generic SGI dispatch optimisation problem. Its aim is to assign resources of $|\mathcal{P}_s|$ service suppliers to $|\mathcal{P}_k|$ customers, where $\mathcal{P}_s$ and $\mathcal{P}_k$ are the sets of suppliers and customers accordingly. Firstly, to democratically consider all participants in the dispatch process, with respect to their preferences, we suggest to allocate criteria on a per dispatch participant basis rather than considering a few global ones. The participants' preferences and requirements should be either directly provided by them or derived automatically from renowned application-specific guidelines or expert knowledge. We advise that each participant has a set of cost and quality criteria associated with them formulated according to physical aspects of the considered SGI system. By *cost* we understand any criteria undergoing minimisation, not necessarily monetary ones. This may also include time-specific criteria. Secondly, to assure that the dispatch is performed, we include variables associated with each dispatchable resource. Those may either be discrete or continuous — depending on specific SGI applications. To ensure that the dispatch decisions allow for covering the participants' demands for the service, specific balancing constraints must be added. Finally, to make technically-feasible decisions the dispatch optimisation must happen on the feasible region, defined through specific constraints. Those should model as precisely as possible the current operational state of the system as known to the dispatcher at the decision making stage.

The designed problem falls into the category of multi-criteria optimisation. To ensure appropriate treatment of all dispatch participants it must be solved so that the Pareto-optimality or in some cases even equity of the solution is guaranteed. The results to the problem under these two assumptions are further investigated in this work in Chapter 4. It is then up to the Decision Maker or the Legislator to decide if they want to assure the equity of the solution or not.

1.5 Conclusions

In this Chapter we introduced the importance of SGIs to the well-being of the Europeans. We have outlined that the operation of SGIs is a complex, multi-stage task, parts of which can be facilitated by computer-based DSSs, and more precisely with mathematical optimisation, which were described in more details. We also described the notion of *quality* and how it fits in the dispatching of the SGIs. We enlisted a few possible SGIs, where integration of the quality-based criteria in the dispatching process may bring benefits to the dispatching results. Having said the above, we identified the research thesis with the supporting research objectives and concluded the Chapter by the high-level design of the generic SGI dispatch optimisation problem with quality-based criteria.

From the information analysed within this Chapter, we may conclude that this work falls within the research trends on hybrid Decision Support Systems as applied to the multi-criteria dispatch of the SGIs. We formulate and study mathematical optimisation models, which take preferences/requirements either from the participants' directly or from renowned application-specific guidelines/expert knowledge. Hence, we combine *model-driven* with *knowledge-driven* DSSs, creating a hybrid of those. We aim to provide outcomes, which in future could possible be incorporated into such hybrid DSSs. The graphical representation of the relation on the area where this work lands with respect to the research on DSSs is given in the Venn diagram in Fig. 1.2. The research area of this work is shown as the red dot.

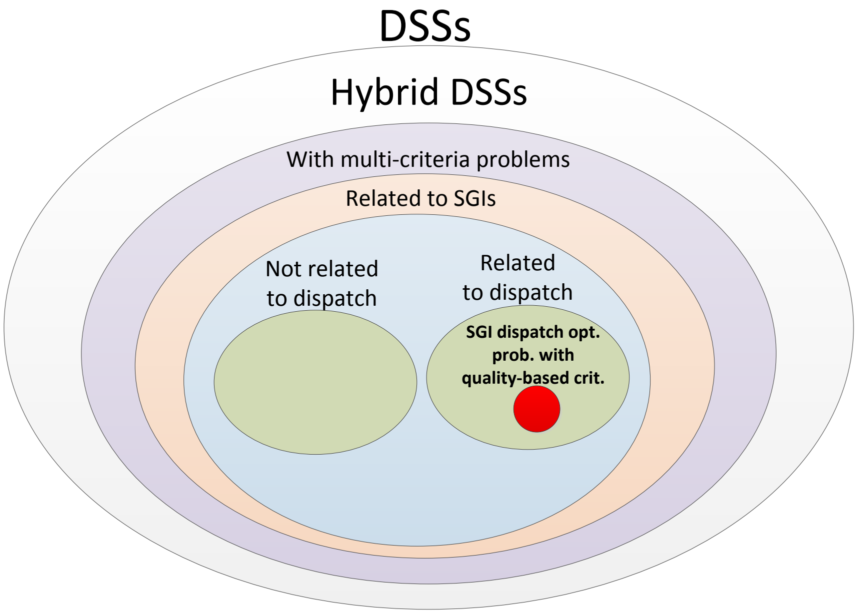


Figure 1.2: Relation between DSSs and the research area of this work

Chapter 2

Review of the concepts

In this Chapter we investigate the concepts related to the statement of the generic SGI dispatch optimisation problem — both from the general operational research and specific application problems perspective. Then, as the problem proposed is a multi-criteria one, we then describe different notions related to such optimisation.

2.1 Work related to the problem statement

This Section provides an overview of the related work found in the literature from a general operational research perspective. Pieces of the research related to more specific applications presented in the Case Studies are discussed in the respective *Literature review* Sections directly in the Case Study Chapters.

From the SGI problem statement perspective, this work is related to the *Generalised Assignment Optimisation* problem [64], and more specifically, to its multi-criteria formulation [65, 66, 67]. These problems aim to answer the question of how to best assign n tasks to n available machines or, in other words — n suppliers to n customers. Its multi-criteria formulation presented in [67] is shown in (2.1):

$$
\begin{aligned}
\max \quad & [\sum_{i=1}^{n}\sum_{j=1}^{n} p_{i,j}^{r} t_{i,j}, \quad -\sum_{i=1}^{n}\sum_{j=1}^{n} c_{i,j}^{m} t_{i,j}] \quad \forall r = 1, \ldots, s, \forall m = 1, \ldots, k \\
\text{s.t.} \quad & \sum_{j=1}^{n} t_{i,j} = 1 \qquad \forall i = 1, \ldots, n, \\
& \sum_{i=1}^{n} t_{i,j} = 1 \qquad \forall j = 1, \ldots, n, \\
& t_{i,j} \in \{0; 1\} \qquad \forall i, j = 1, \ldots, n
\end{aligned}
\tag{2.1}
$$

where:

- $t_{i,j}$ — variable representing the assignment of task/customer i to machine/supplier j;

- $c_{i,j}^m$ — cost parameter for m^{th} minimised criterion;
- $p_{i,j}^r$ — profit parameter for r^{th} maximised criterion;
- k — number of minimised criteria;
- s — number of maximised criteria.

There have been some significant research developments in the field of multi-criteria general assignment problems. Apart from applying standard general-purpose scalarisation techniques as discussed in [68, 69] and in further chapters, some researchers propose to solve it by using the Data Envelopment Analysis (DEA) [67, 70]. In addition to the above, other dedicated interesting solution developments have been proposed. Belhoul et al. [71] propose a method making use of the *k-best* algorithm for finding the best compromise solutions. Another dedicated algorithm was proposed by Przybylski et al. [72], who tackled the problem by a two-stage method — first by looking at supported efficient solutions, and then at the search area inside of which non-supported efficient points may exist. What is more, this problem was also successfully tackled by means of some metaheuristics [73, 74].

The problem (2.1) has also been put into multiple various applications, sometimes extending or amending the basic formulation to fit the desired purpose. One of the interesting applications of the extension of (2.1) is the *Gate Assignment Problem*, which aims to assign aeroplanes to gates at a given airport. According to the review study performed by Daş et al. [75], in recent years, more focus has been put into considering multiple criteria, with criteria usually oriented towards passengers. Yet, there exist formulations that look into the criteria of the airport and the airline (e.g., as in the work by Kaliszewski et al. [76]).

Apart from the above, an extension of (2.1) can be found in the *Task Assignment* problems, where a group of agents shall be assigned to a group of tasks. Those are used to assign workers with differing skills to tasks [77, 78]. Its specific version is assigning military personnel to missions taking into consideration the cost of assignment and the *suitability* of personnel to the mission [79]. A similar topic was also studied by Ferguson et al. [80], whose study considered four different objectives — type of assignment, officers' preferences, the overall assignment preference satisfaction and the solution stability (which is not used in their mixed-integer linear programming formulation). Such multi-criteria assignment problems were also studied in applications to healthcare by B. Sawik in his multiple works [81, 82, 83]. Author developed and solved using multiple methods a model to assign employees to different jobs or services of a real hospital (namely the Rydygier Specialist Hospital in Kraków, Poland), taking into account the available budget and requirements of each job. According to Sawik, such models can well be solved by the reference point, lexicographic and weighted-sum aggregation methods. More information on those methods is given in Sec. 3.4 of this work.

The multi-criteria formulations of this problem are often used in the assignment of tasks for unmanned aerial vehicles (UAVs). Song et al. [84] proposed a model for joint UAV station location and UAV task optimisation problem that optimised three criteria — minimisation of the total travelling time of UAVs, minimisation of the total station placement cost and maximisation of the total number of tasks assigned. Another paper that tackled a similar application is [85], where the authors looked at assigning tasks to military UAVs in battlefield combat considering two criteria. Some other interesting works in this field include [86, 87].

We can also find some notable pieces of research using the multi-criteria assignment problem for transportation in applications like ride matching [88] or city bikes dispatch to stations [89].

Similar problem in its nature has also been developed by T. Sawik [90], where cyber security controls were selected to protect the supply chains in Industry 4.0. The selection was computed in order to minimise both the security investment capital cost and possible financial losses in case of a cyber incident.

One should notice that one set of criteria in (2.1) is maximised and the other is minimised. We apply a similar approach in our proposed problem (3.1). Yet, in the proposed problem, we give specific meaning to the maximised criteria — namely, we link them to the quality of service received by the customer. What is more, we assign criteria for each and every dispatch participant to specifically consider the individual service they receive rather than treating the criteria in a global way. This is discussed in greater detail in the next Chapters.

Although in the problem (2.1), as cited directly from [67], it is considered that the number of available machines is equal to the number of tasks — n. However, this could be lifted in a more general formulation, and a different number of those can be considered.

What is more, in a more general formulation, one may allow $t_{i,j}$ to take real values and allow suppliers to serve multiple customers (or machines to serve multiple tasks) such that the demand is covered. In that way, one will face the balancing problem, sometimes known as *Economic Dispatch*. Its simple formulation is given in Toczyłowski [47], however, in a single-criterion form. Thus, in this Section, we show the problem with constraints as given in [47], but with similar criteria as in (2.1) and we consider variables to be two-dimensional. This represents the amount of resource supplied by machine/supplier j to customer/task i and is done to give more flexibility to the formulation. Similarly, the demand is considered on a per-customer basis. The modified formulation is given in (2.2):

$$
\begin{aligned}
\max \quad & [\sum_{i=1}^{n}\sum_{j=1}^{n} p_{i,j}^{r} t_{i,j}, \quad -\sum_{i=1}^{n}\sum_{j=1}^{n} c_{i,j}^{m} t_{i,j}] \quad \forall r = 1, \ldots, s, \forall m = 1, \ldots, k \\
\text{s.t.} \quad & \sum_{j=1}^{n} t_{i,j} = \Delta_i \qquad \forall i = 1, \ldots, n, \\
& \underline{t_{i,j}} \leq t_{i,j} \leq \overline{t_{i,j}} \qquad \forall i, j = 1, \ldots, n
\end{aligned} \tag{2.2}
$$

where:

- Δ_i — demand for service raised by customer/task i (parameter);
- $\underline{t_{i,j}}$ / $\overline{t_{i,j}}$ — minimum/maximum supply capabilities from j to i (parameters).

It is worth noting that depending on the situation, the second constraint could possibly be substituted by the *sum constraint*: $\underline{t_i} \leq \sum_{i=1}^{n} t_{i,j} \leq \overline{t_i}\ \forall j = 1, \ldots, n$ to represent the fact that suppliers' capabilities are specified for all customers jointly and not for each and every precise one. However, we decided to show a more general formulation in (2.2) with the possibility to restrict those on a per-customer basis.

Problem (2.2) is widely studied in the dispatch of units generating electrical energy so that the electrical demand of customers is met. One should note that some criteria considered in the study of electrical energy are sometimes non-linear [47, 91]. However, it is not relevant to the further Sections of our study, and thus (2.2) is presented in the linear form.

Multi-criteria economic dispatch is of vast research interest. This can be seen in the notable review papers [92, 93]. Jubril et al. [94] proposed to solve the problem with two criteria — minimisation of the total generation cost and minimisation of the distribution losses. They also made a proposition to solve the problem using Semidefinite Programming (SDP). The topic was also taken by Hou et al. [95], who considered in their model the existence of transferable load and electric vehicles (EVs). They considered three optimisation criteria — economic efficiency, power system security and efficiency of operations. EVs and vehicle-to-grid technology were also considered in [96], where Authors considered the minimisation of generation cost, minimisation of carbon dioxide emissions and minimisation of pollutant treatment cost. The addition of environmental criteria into the dispatch is also a topic of high research interest, which is reflected in the following papers [97, 98, 99]. It is worth noting that in many of the references cited in this Section, various, often non-linear criteria are used. This is because the researchers aim to model the complex cost associated with energy generation. However, this does not compromise the formulation (2.2) but simply extends it.

Multi-criteria assignment and dispatch is a topic of extensive research in recent years. We have shown in this Section that not only have these models been successfully applied to some instances of the Services of General Interest (airport gate assignment, ride matching, electrical energy generation), but also to the assignment of military personnel to missions and to dispatch of UAVs.

However, to the best of our knowledge, we were not able to identify research works, which are devoted to more general multi-criteria optimisation models for dispatch of SGIs, or of public services. Those identified were always problem-specific. The only papers, we were able to identify that apply operational research methods to general public services are [100, 101, 102]. Yet, Ansari et al. [100] developed a hypercube spatial queuing model for multiple servers to be located in the same station and then dispatched to the same call, which is more of a simulation model. They do not embed it in any dispatch optimisation problem, although it might be a good idea. Da Silva et al. [101], however, propose to use linear programming in defining optimal tariffs for public services. Marianov et al. [102] studied the problems of location and not dispatch. Therefore, the above-mentioned papers are not directly linked to the problem of SGI dispatch considered in this work. However, an interesting paper in that field was published by Swersey [103], who reviewed the literature related to deployment optimisation problems for emergency services (EMS, Fire Brigade and Police) jointly. Yet, he did not analyse in his work other classes of SGIs, such as, for instance, electrical energy supply.

None of the papers related to the assignment/dispatch problems identified in the course of this survey consider quality as one of the criteria, in a way that its value is requested directly by the participants. What is more, usually multiple criteria in the dispatch are considered globally and not on a per-participant basis. This means that we have a couple of aggregate criteria established for the whole system and not a couple of individual criteria linked to each and every participant.

Therefore, in this work, we intend to fill the gaps identified. We propose a generic multi-criteria SGI dispatch optimisation problem with quality-based criteria, which can be applied to many different services, provided that they are dispatchable and differentiable on the type of service offered. What is more, we consider criteria on a per-participant basis, where one set of them is specifically related to the quality of service received by the customer. As shown in the next chapters, by considering each participant's criteria separately, we aim to consider their specific needs towards the service received.

As already mentioned, this review is focused only on the general concepts applicable to this work. Thus, we focused on the general problem statements. Specific literature related to Case Studies is reviewed in the respective Chapters. There, we also discuss the classical approaches to dispatching of those services used today and compare them with the proposed approach. Those are not given in this Section, as we were unable to identify such general papers applicable to multiple different SGIs.

2.2 Multi-criteria optimisation

This Section presents the theoretical concepts behind this class of the optimisation problems. Apart from setting theoretical grounds for the multi-criteria optimisation, we discuss the implications of different concepts and methods to the generic SGI dispatch optimisation problem with quality-based criteria. Some basic definitions related to the binary relations referred to in this Section are given in Appendix A.

2.2.1 Preferences and efficiency

This Section gives a notion of the preference model and the efficiency of the solution. These concepts formulate the core of any multi-criteria optimisation solution strategies. For the sake of conceptual simplification, without loss of generality, let us denote a multi-criteria optimisation problem in a compact way (2.3), where $\mathcal{Q}$ is the feasible region and $\boldsymbol{x}$ the vector of decision variables.

$$\begin{aligned} \max \quad & \boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_p(\boldsymbol{x})] \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{Q} \end{aligned} \tag{2.3}$$

The fundamental constitution of any multi-criteria analysis, namely *Pareto-optimality*, also known as *efficiency* or *non-dominance* of the solution. Note, that we describe those concepts for the maximisation problem.

Definition 2.2.1 (Pareto-optimality / efficiency / non-dominance). A feasible decision variable vector $\boldsymbol{x}^*$ is a *Pareto-optimal* or *efficient*, or *non-dominated* solution of the problem (2.3), iff $\nexists\ \boldsymbol{x}^0 \in \mathcal{Q}, \boldsymbol{x}^0 \neq \boldsymbol{x}^*$, such that $\boldsymbol{f}(\boldsymbol{x}^*) \leq \boldsymbol{f}(\boldsymbol{x}^0)$, where at least one strict inequality holds [104, 105]. In case this is not true (no strict inequality holds), such a solution is called *weakly efficient* [106].

As can be concluded from the definition and from the nature of the problem, usually there are multiple Pareto-optimal solutions of the multi-criteria optimisation problems. The multiple efficient solutions form the so-called *Pareto front* or *Pareto set*. These solutions are referred to as *trade-off solutions*, as in a general situation of contradicting criteria, improving one criterion will result in the worsening of another. Therefore, the standard goal of multi-criteria optimisation is to generate a number of Pareto-optimal solutions, which are to be presented to the Decision Maker (DM). Ultimately, the DM will make up their mind and choose a solution which suits their needs best.

Now, let us look at some examples of the solutions of (2.3) with $p = 6$, i.e., $\boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), f_3(\boldsymbol{x}), f_4(\boldsymbol{x}), f_5(\boldsymbol{x}), f_6(\boldsymbol{x})]$. Exemplary solutions under consideration are shown in Tab. 2.1.

Table 2.1: Exemplary solutions of a multi-criteria problem

Exemplary solutions of (2.3)						
	Values of criteria					
Solution	f_1	f_2	f_3	f_4	f_5	f_6
S1	0	-2	0	-2	0	-2
S2	1	-5	1	0	1	-2
S3	1	-2	0	-2	0	-2
S4	3	-1	3	-1	3	-2

One can identify that solutions S4 and S2 are the only Pareto-optimal ones. Solution S1 is dominated by S3 and S4, and S3 is dominated by S4.

It is however questionable whether both S2 and S4 are good enough to be directly used for the dispatch in the problem considered. One should remember that SGIs should be accessible to all EU citizens and that their main foundation principle is the enhancement of the well-being of the Europeans. Having said the above, we introduce the concept of *equitable efficiency* of the solutions [105, 107].

2.2.2 Equitable efficiency

When comparable criteria are considered, it is possible to generate solutions which are *equitably efficient* or, in other words, *fair*. In this Section, we discuss this concept and provide some insights into the SGI dispatch optimisation problem. Note, that we describe those concepts for the maximisation problem.

First, let us clarify some notation used in this Section. Let "$\preceq$" be a preference relation in $\mathbb{R}^p$.

Definition 2.2.2 (Impartial relation [107, 108]). Relation $\preceq$ is *impartial*, if

$$[f_{\pi(1)}(\boldsymbol{x}), ..., f_{\pi(p)}(\boldsymbol{x})] \cong [f_1(\boldsymbol{x}), ..., f_p(\boldsymbol{x})],$$

where $\pi \in \Pi$ and $\Pi = \{\pi | \pi \text{ is any permutation of the set of indices } I = \{1, 2, \ldots, p\}\}$ and "$\cong$" expresses the relation of *indifference*.

Definition 2.2.3 (Indifference [108]). Relation of *indifference* can be related to $\preceq$ as

$$[f_{\pi(1)}(\boldsymbol{x}), ..., f_{\pi(p)}(\boldsymbol{x})] \cong [f_1(\boldsymbol{x}), ..., f_p(\boldsymbol{x})] \text{ iff}$$

$$[f_{\pi(1)}(\boldsymbol{x}), ..., f_{\pi(p)}(\boldsymbol{x})] \preceq [f_1(\boldsymbol{x}), ..., f_p(\boldsymbol{x})] \text{ and } [f_1(\boldsymbol{x}), ..., f_p(\boldsymbol{x})] \preceq [f_{\pi(1)}(\boldsymbol{x}), ..., f_{\pi(p)}(\boldsymbol{x})]$$

for any permutation $\pi \in \Pi$ of indices.

Definition 2.2.4 (Pigou — Dalton principle of transfers [105, 108, 109]). Relation "$\preceq$" satisfies the *principle of transfers* if it fulfils the below axiom:

$$f_i(\boldsymbol{x}) > f_j(\boldsymbol{x}) \implies \boldsymbol{f}(\boldsymbol{x}) - \epsilon \boldsymbol{e}_i + \epsilon \boldsymbol{e}_j \prec \boldsymbol{f}(\boldsymbol{x}) \text{ for } 0 < \epsilon < f_i(\boldsymbol{x}) - f_j(\boldsymbol{x})$$

where $\boldsymbol{e}_i \in \mathbb{R}^p$ is a unit vector, whose i^{th} component equals 1.

In other words, the principle of transfers states that: A transfer of any small amount from an outcome vector to any subjectively worse outcome vector will result in a more preferred outcome vector.

Having defined the basic notation and principles behind equitable preference and efficiency, we can now proceed to define the *equitable* or *fair efficiency*.

Definition 2.2.5 (Equitable/fair preference [107, 108]). A relation "$\preceq_e$" defined on $\mathbb{R}^p$ is an *equitable/fair preference relation* if it is reflexive, transitive, strictly monotonic, impartial and satisfies the principle of transfers.

Definition 2.2.6 (Equitable or fair efficiency / non-dominance). Any feasible decision variable vector $\boldsymbol{x}_e^*$ is an *equitably/fair efficient/non-dominated* solution of the problem (2.3)), iff $\nexists\, \boldsymbol{x}^0 \in \mathcal{Q}, \boldsymbol{x}^0 \neq \boldsymbol{x}_e^*$, such that $\boldsymbol{f}(\boldsymbol{x}_e^*) \preceq_e \boldsymbol{f}(\boldsymbol{x}^0)$.

To effectively use the concept of equitable efficiency in multi-criteria optimisation let us introduce the $\boldsymbol{\Theta}$ and $\overline{\boldsymbol{\Theta}}$ mapping operators [105].

Definition 2.2.7 (Mapping $\boldsymbol{\Theta}$). Let $\boldsymbol{\Theta} : \mathbb{R}^p \mapsto \mathbb{R}^p$ be a mapping such that
$\boldsymbol{\Theta}(\boldsymbol{f}(\boldsymbol{x})) = [\theta_1(\boldsymbol{f}(\boldsymbol{x})), ..., \theta_p(\boldsymbol{f}(\boldsymbol{x}))]$, where $\theta_1(\boldsymbol{f}(\boldsymbol{x})) \leq \theta_2(\boldsymbol{f}(\boldsymbol{x})) \leq ... \leq \theta_p(\boldsymbol{f}(\boldsymbol{x}))$ and there exist a permutation π of the set of indices $I = \{1, 2, ..., p\}$ such that $\theta_k(\boldsymbol{f}(\boldsymbol{x})) = f_{\pi(k)}(\boldsymbol{x})$. In other words, $\boldsymbol{\Theta}$ is an operator that orders elements of $\boldsymbol{f}(\boldsymbol{x})$ in a non-decreasing order.

Definition 2.2.8 (Mapping $\overline{\boldsymbol{\Theta}}$). Let $\overline{\boldsymbol{\Theta}} : \mathbb{R}^p \mapsto \mathbb{R}^p$ be a mapping defined such that $\overline{\boldsymbol{\Theta}}(\boldsymbol{f}(\boldsymbol{x})) = [\overline{\theta_1}(\boldsymbol{f}(\boldsymbol{x})), ..., \overline{\theta_p}(\boldsymbol{f}(\boldsymbol{x}))]$, where $\overline{\theta_k}(\boldsymbol{f}(\boldsymbol{x})) = \sum_{j=1}^{k} \theta_j(\boldsymbol{f}(\boldsymbol{x}))$. In other words, $\overline{\theta_k}(\boldsymbol{f}(\boldsymbol{x}))$ expresses the sum of k smallest outcomes.

Once introduced, we can now leverage the $\overline{\boldsymbol{\Theta}}$ operator to identify the equitable dominance of solutions.

Theorem 2.2.1. Solution $\boldsymbol{x}^1$ equitably/fairly dominates solution $\boldsymbol{x}^0$, iff
$\overline{\theta_k}(\boldsymbol{f}(\boldsymbol{x}^1)) \geq \overline{\theta_k}(\boldsymbol{f}(\boldsymbol{x}^0))\ \forall k = 1, 2, ..., p$ where at least one strict inequality holds [110, 105].

The proof of Theorem 2.2.1 can be found in [105] and will be omitted here. It is worth noting, however, that by definition, this Theorem together with Definition 2.2.8 allow us to conclude that any Pareto-optimal solution of the multi-criteria problem (2.4) is an equitably efficient solution of problem (2.3).

$$\begin{aligned} \max \quad & \boldsymbol{z} = [z_1, z_2, \dots, z_p] \\ \text{s.t.} \quad & z_k = \overline{\theta_k}(\boldsymbol{f}(\boldsymbol{x})) \qquad \forall k = 1, \dots, p, \\ & \boldsymbol{x} \in \mathcal{Q} \end{aligned} \tag{2.4}$$

One should also note that any efficient solution of (2.4) is a Pareto-optimal solution of (2.3). However, not all Pareto-optimal solutions of (2.3) are equitably efficient — i.e., are not efficient solutions of (2.4). Proof of this statement may be found in [110].

Unfortunately the problem (2.4) may be difficult in direct implementation due to the $\overline{\theta_k}(\boldsymbol{x})$ component. However, Ogryczak et al. [110] propose to deal with this problem by leveraging the dual Linear Programme (LP) to (2.4). Once done, the problem is equivalent to (2.5), which is a standard LP programme. Reader interested in the reformulation is advised to consult the source paper directly.

$$\begin{aligned} \max \quad & \boldsymbol{z} = [z_1, z_2, \dots, z_p] \\ \text{s.t.} \quad & z_k = kt_k - \sum_{i=1}^{p} d_{i,k} \qquad \forall k = 1, \dots, p, \\ & t_k - d_{i,k} \leq f_i(\boldsymbol{x}) \qquad \forall k, i = 1, \dots, p, \\ & \boldsymbol{x} \in \mathcal{Q} \end{aligned} \tag{2.5}$$

In a general case of multi-criteria optimisation or analysis, the criteria are deemed incomparable [108]. This, however, is only partially true for the case of SGI dispatch optimisation problem. In this problem criteria are associated with the dispatching process participants, whose criteria are pair-wise similar (cost/quality). In that sense, they are pair-wise comparable. For instance, if the example given in Tab. 2.1 represented the solution of SGI dispatch, one could compare f_1 with f_3 and f_5, as they would represent the quality criteria associated with three dispatch participants. Similarly, f_2 could be compared with f_4 and f_6 as they would represent the associated cost criteria. This is a general property of the generic SGI dispatch optimisation problem with quality-based criteria. Although not all criteria are comparable in the problem considered, it is still possible to make them comparable by applying some properly crafted *achievement functions* [111]. Therefore, it is valid to assume that techniques generating equitably efficient solutions can be used in the SGI dispatching procedure.

In our example presented in Tab. 2.1, the only equitably efficient solution is S4. This is clearly visible when $\boldsymbol{\Theta}$ and $\overline{\boldsymbol{\Theta}}$ operators are applied. As already stated, only Pareto-optimal solutions of the original problem (2.3) are candidates to be the equitably efficient solutions of the same. Therefore, we will only show the results of $\boldsymbol{\Theta}$ and $\overline{\boldsymbol{\Theta}}$ for solutions S2 and S4. For solution S2 the $\boldsymbol{\Theta}_{S2} = [-5, -2, 0, 1, 1, 1]$ and respectively $\overline{\boldsymbol{\Theta}_{S2}} = [-5, -7, -7, -6, -5, -4]$. For S4, however they equal $\boldsymbol{\Theta}_{S4} = [-2, -1, -1, 3, 3, 3]$ and $\overline{\boldsymbol{\Theta}_{S4}} = [-2, -3, -4, -1, 2, 5]$. As can be seen, $\overline{\boldsymbol{\Theta}_{S4}} > \overline{\Theta_{S2}}$ for all elements of the vectors. Hence, according to the Theorem 2.2.1, S4 equitably dominates S2 and we can conclude that S4 is the only equitably efficient solution in the example presented.

2.3 Summary

In this Chapter we reviewed the concepts applicable to the generic SGI dispatch optimisation problem with quality-based criteria. First, we reviewed the literature related to the possible problem statement. In the literature reviewed we have not found any research work that proposed any specific generic SGI dispatch optimisation problem. What is more, none of the papers have considered inclusion of quality-based criteria to the dispatch on a per-participant basis.

After this, we reviewed the general concepts related to the multi-criteria optimisation, as the problem considered is of this type. In the review of general concepts of the multi-criteria optimisation we focused on reminding the reader of definitions of Pareto-optimality and equity (or fairness) of the solution. These concepts are crucial for the dispatch of SGIs.

SGIs are meant to improve the life of *all* European Union citizens. Therefore, in the dispatch of SGIs, one should never consider dominated solutions, as some of their criteria can be improved without worsening any of the others. Hence, dispatch support procedures should only generate Pareto-optimal solutions. In a normal multi-criteria analysis situation, they would be presented to the Decision Maker to make the final choice between them. Yet, in the case of SGI dispatch, where the decision by the dispatcher must be made as soon as possible to allow for the timely provision of the services, one should not spend too much time to generate multiple Pareto-optimal solutions for comparison. Thus the first solution produced by the optimiser may be considered valid and implemented straight away, provided that it responds to the needs of the participants.

Chapter 3

Generic SGI dispatch optimisation problem with quality-based criteria

In this Chapter we formulate the generic multi-criteria Services of General Interest dispatching optimisation problem with quality-based criteria. Firstly, we state the problem mathematically and describe the assumptions made. Secondly, we analyse the problem proposed functionally as opposed to the already presented requirements. After setting this ground, we describe the different solution strategies, all with respect to the proposed SGI dispatch optimisation problem.

3.1 Mathematical formulation

This Section gives the mathematical formulation of the generic multi-criteria Services of General Interest dispatching optimisation problem with quality-based criteria in (3.1). In the remainder of this work we do refer to it using also the acronym: **SDOQ** (SGI **D**ispatch **O**ptimisation problem with **Q**uality-based criteria). The formulation is derived from observations and design requirements given in Chapter 1. The aim is to dispatch the resources in possession of service suppliers from the set $\mathcal{P}_s$ to the customers from the set $\mathcal{P}_k$ over a given time horizon T. The dispatch is organised in a way that the demand of each customer i in time instance t, denoted as Δ_i^t, is covered. The members of the subset of the union of the suppliers' and customers' sets, whose preferences are taken into consideration in the optimisation process are called *dispatch participants*. For each participant, we define a set of quality and cost criteria, formulation of which should be made by a subject matter expert in the domain, to which the SDOQ problem (3.1) is to be applied. In this formulation they are presented as a certain functions of the decision variables. We do not specify their exact formulation as it is specific to a given application. Quality criteria are maximised, whereas cost ones are minimised. It should be noted, that the name *cost* is only a certain convention we apply and is not linked to any monetary aspects. The cost criterion can very well represent for example time. We simply call it this way, as it undergoes minimisation. As in the problem (2.2), the constraints given assure covering of demand for the service dispatched on a per-customer basis. The dispatch of resource from supplier j to customer i in time instance t is modelled with the help of the $P^t_{i,j}$ variables. Depending on the exact dispatch situation those can be either continuous or integer. This variable is bounded by each supplier's maximum supply capability to customer i defined by the parameter $\overline{P^t_{i,j}}$. In this formulation we differentiate capabilities on a per-customer basis, yet those can also be bounded by the overall maximum (if differentiation is not needed). In this situation a constraint in the form of $\sum_{i \in \mathcal{P}_k} P^t_{i,j} \le \overline{P^t_j}\ \forall j, t$ should substitute the fourth constraint in the formulation. The dispatch decisions must belong to the feasible set $\mathcal{Q}$, which is to be defined on a per-application basis by a subject matter expert and expressed in the form of specific constraints, possibly with auxiliary variables.

SDOQ:

$$
\begin{aligned}
\max \quad & [q_{1,1}, -c_{1,1}, q_{1,2}, -c_{1,2}, \dots, q_{\overline{m},\overline{n}}, -c_{\overline{m},\overline{k}}] \qquad \overline{m} = |\mathcal{P}|, \overline{n} = |\mathcal{I}|, \overline{k} = |\mathcal{K}| \\
\text{s.t.} \quad & q_{m,n} = f^1_{m,n}(\boldsymbol{x}) \qquad \forall m = 1, \dots, \overline{m} \quad \forall n = 1, \dots, \overline{n}, \\
& c_{m,k} = f^2_{m,k}(\boldsymbol{x}) \qquad \forall m = 1, \dots, \overline{m} \quad \forall k = 1, \dots, \overline{k}, \\
& \sum_{j=1}^{|\mathcal{P}_s|} P^t_{i,j} = \Delta^t_i \qquad \forall i = 1, \dots, |\mathcal{P}_k| \quad \forall t = 1, \dots, T, \\
& 0 \le P^t_{i,j} \le \overline{P^t_{i,j}} \qquad \forall i = 1, \dots, |\mathcal{P}_k| \quad \forall t = 1, \dots, T \quad \forall j = 1, \dots, |\mathcal{P}_s|, \\
& \boldsymbol{x} \in \mathcal{Q}
\end{aligned}
\tag{3.1}
$$

where:

- $q_{m,n}$ — n^{th}quality criterion of participant m, being function f^1 of $\boldsymbol{x}$, criteria maximised (variable);
- $c_{m,k}$ — k^{th} cost criteria of participant m, being function f^2 of $\boldsymbol{x}$, criteria minimised (variable);
- $\boldsymbol{x}$ — vector of decision variables, composed of $P^t_{i,j}$ as its elements;
- $P^t_{i,j}$ — amount of *service* delivered by supplier j to customer i in time instance t (variable);
- Δ^t_i — demand for the service issued by customer i in time instance t, for assignment problem should be equal 1 (parameter);
- $\overline{P^t_{i,j}}$ — maximum supply capabilities of supplier j to customer i in time instance t (parameter);

- $\mathcal{Q}$ — feasible set of $\boldsymbol{x}$ describing the bounds on decision variables imposed by the physical limits. [1]

- $\mathcal{P}_s$ — set of suppliers;

- $\mathcal{P}_k$ — set of customers;

- $\mathcal{P} \subseteq P_s \cup P_k$ — set of participants whose criteria are considered in the dispatching process;

- $\mathcal{I}$ — set of considered quality criteria indices;

- $\mathcal{K}$ — set of considered cost criteria indices;

- T — number of time instances considered.

The presented formulation is general enough to represent many possible dispatching jobs. There might be various contexts of what the $P^t_{i,j}$ variables represent and whether any additional constraints are imposed on them. Depending on the above, the proposed formulation can be applied to both balancing and assignment problems. For balancing problems, the discussed variables should be continuous, and for the assignment, they should be binary, i.e., $P^t_{i,j} \in \{0, 1\}$. There also might exist applications where they should be generally integer — not necessarily binary.

3.2 Assumptions

The proposed model may be applicable only to selected SGIs. The generic model is intended to facilitate dispatchers' decisions by providing specific dispatch recommendations. In that sense, any policy-making, location or other optimisation problems are deemed out of the scope of this work, even though they all contribute to the overall dispatching problems.

The proposed model is also applicable only to the classes of SGIs, which offer services that can be differentiated based on quality (and/or speciality) they offer. Some examples of those were already presented in the Introduction Chapter.

Another important assumption that we make is that at all times, the number of suppliers is at least as large as the number of customers. [2] This implies that queuing models are not necessary to be considered in this work. They could possibly be incorporated into more complicated models, yet such complication will not bring benefit in responding to the research question posed. Thus,

[1] The set $\mathcal{Q}$ is to be defined on a per-application basis by means of specific constraints, possibly together with auxiliary variables.

[2] Mathematically it means that $|\mathcal{P}_s| \geq |\mathcal{P}_k|$.

it is considered out of scope of this work. An interested reader is advised to consult *queuing theory* references (e.g., [112], where authors apply queuing theory to scheduling of Police patrols, or [113] where it is applied to EMS dispatching). [3]

In this work, we consider only linear programming (LP) and mixed-integer linear programming (MIP) models. Thus we require that $f^1_{m,n}(\boldsymbol{x})$ and $f^2_{m,k}(\boldsymbol{x})$ are modelled as (mixed-integer) linear. However, for other classes of optimisation problems considered or studied, this constraint may be lifted.

3.3 Functional analysis

In this Section we analyse the proposed optimisation model SDOQ (3.1) theoretically from the perspective of satisfying the functional requirements presented in Sec. 1.4. Firstly, in the generic problem each participant is assigned its set of quality and a set of cost criteria for the optimisation of dispatch decisions. Since they are associated with each and every participant, their preferences on the dispatch can be taken into consideration by the optimiser and eventually by the dispatcher of the service. Specifically, the problem does not rely entirely on overall system-wide criteria, which can sometimes disfavour some participants. However, if some general measures are deemed required by the subject matter expert, they can be integrated well.

It is also foreseen that the exact formulation of the cost and quality may differ between SGI applications. This is why we deliberately do not specify their formulation in the generic SDOQ problem (3.1) but leave it to the subject matter experts. Such an approach permits to allocate SGI resources considering the fact that they often differ in the quality of service they offer. In this way, we argue they can be assigned to respond better to participants' exact functional needs.

The formulation allows for considering multiple criteria of each kind to allow for adaptation to various SGI applications. Depending on the solution strategy applied to it, Pareto-optimality or equity of the result may be obtained, regardless of the generic formulation given. This is then in line with the principle of guaranteeing the democratic choices of the dispatch participants. Equity of the result can be achieved if formulation (2.5) is used for this purpose.

Moreover, the generic formulation can be applied to both assignment and balancing problems, depending on whether integrity constraints are imposed on the decision variables or not. It is also foreseen in the formulation to put additional problem-specific constraints. In that way,

[3] This assumption does reflect the reality of at least some SGI systems, such as the Emergency Medical Services or the electrical energy generation and supply. According to the Warsaw Office of Statistics (*Urząd Statystyczny w Warszawie*), in 2021 on average, only around 24% of available EMS units were busy per hour in the whole of Mazowieckie voivodeship, Poland [114]. Also, the electrical energy generation systems are usually designed in a way that they have sufficient capabilities to cover the peak demand. Thus, one may conclude that it is a matter appropriate design.

specific decision situations can be modelled to ensure that the decision made are always technically feasible in the current operational state of the system, as given in Chapters 5 and 6.

3.4 Analysis of multi-criteria problem solution techniques

The problem SDOQ (3.1) is a multi-criteria one. Therefore, to solve it, some specific techniques should be applied. In this Section, we give and compare some of the known solution strategies with respect to the problem under consideration. Unless clearly specified in the respective Section — e.g., maxi-min aggregation, all of the methods discussed are proven to produce Pareto-optimal solutions. For simplicity, and without loss of generality, we can assume that the generic multi-criteria problem (2.3) with its equitable version (2.5) are a more compact and generalised notations of the generic SGI dispatching optimisation problem with quality-based criteria SDOQ (3.1).

In the remainder of this Section, we show aggregations applied to the problem (2.3). Yet, some of them can be directly applied to the (2.5) as well. This Section gives a non-extensive list of possible scalarising techniques, as a detailed investigation of different techniques does not constitute the main research objective of this work. An interested reader is advised to consult [115, 116], where they can find more insights on solution techniques, together with theory on interconnections between them.

3.4.1 General-purpose multi-criteria solvers

For single-criterion optimisation problems a variety of general-purpose, commercial solvers have been developed. Some examples of those are: Gurobi [117], CPLEX [118], MOSEK [119] or Xpress [120]. Commercial solvers like CPLEX, Gurobi or Xpress support directly the specification of multi-criteria problems. However, to the best of our knowledge this is accomplished by implicitly implemented weighted sum scalarisation (see Sec. 3.4.2) or lexicographic optimisation (see Sec. 3.4.4) [121, 122, 123].

Apart from the above, there exist however some academic solvers which allow for enumeration of multiple non-dominated solutions. According to [124] some examples of those are:

- PolySCIP [125] — solver for Multi-Criteria Integer/Linear Programmes;
- Bensolve [126, 127] — solver for Vector Linear Programmes (VLP), in particular, for the subclass of Multi-Criteria Linear Programmes;
- inner [128] — solver for Multi-Criteria Linear Programmes;
- Symphony [129] — solver for Multi-Criteria Bi-objective Integer problems, based on the weighted Chebyshev scalarisation [115].

Apart from already mentioned techniques there exist also some solvers, based on solve heuristics — e.g., [130]. They are however discussed separately in Sec. 3.4.8.

Discussion: The above-listed solvers all have a specific algorithm implemented that either allows for the enumeration of Pareto-optimal solutions or for producing a single one, using one of the scalarisation techniques discussed further in this Section.

Some of the commercial solvers allow for specification of multi-criteria problems. Yet, this is usually implemented by the means of weighted sum scalarisations and of lexicographic maximisation. Weighted sum scalarisation may not be fully suitable for the SGI dispatch optimisation problem (see Discussion in Sec. 3.4.2). Lexicographic optimisation seems to be a better candidate but only to selected problems, namely the equitable ones. However, the exact implementation might be computationally costly. This drawback can be overcome by means of lexicographic approximation (see Sec. 3.4.4). Unfortunately, the implementation of the same may differ on a per-solver basis.

Academic solvers are a powerful tool, yet only for very specific classes of problems. It will not be possible to apply them to any Multi-Criteria Mixed Integer Linear Programme. What is more, usually their main focus is to generate multiple non-dominated solutions to be then evaluated by the Decision Maker. This is not the main goal of solving the SGI dispatch optimisation problem, where we care for generating a single good solution, which could be quickly applied by the dispatcher. It must be noted that the dispatchers would likely operate in a stressful environment, where timely decision counts, and will not be able to evaluate multiple non-dominated solutions calmly (see Discussion in Sec. 3.4.2). Moreover, those solvers usually also make use of either some specific algorithm or of an extended version of a scalarisation technique described further.

All things considered, the solvers are a fairly specific class of solution strategies. As some of them make use of techniques described further, they are not comparable directly with those techniques. Hence, we do not investigate them further and neither do we put them in the comparison table presented at the end of this Chapter.

3.4.2 Weighted sum of criteria

In this Section we cite the mathematical formulation after Ehrgott [131]. This well-known technique groups the criteria considered into a single scalar function by means of assigning positive weights to individual criteria. These weights are assigned basing on *expert knowledge* such that the higher the weight, the more importance is put to optimising the associated criterion. Aggregation of (2.3) by means of the weighted sum is shown in (3.2).

$$
\begin{aligned}
\max \quad & f(\boldsymbol{x}) = \sum_{k=1}^{p} w_k \, f_k(\boldsymbol{x}) \\
\text{s.t.} \quad & \boldsymbol{x} \in Q
\end{aligned}
\tag{3.2}
$$

where $w_k > 0, \forall k = 1, \ldots, p$ — weight associated to the k^{th} criterion

Discussion: The weighted sum aggregation is easy to be implemented and should not introduce any additional important computational burden, as the scalarising function is linear. However, the proper choice of weights is not a trivial task. In the case of SGI dispatch optimisation problem it is rather difficult to justify prioritisation of one participant over another by the means of so-chosen weights. Imagine a situation, where an ambulance has a time-to-arrival priority to patient A over to patient B because the cost criterion of patient A was assigned a higher weight than B. This aggregation can however serve well to solve (2.5) as the components of the goal function are themselves aggregations of the master's problem (2.3) criteria. Thus, the weights would only prioritise the optimisation of the aggregations and not the optimisation of the criteria directly. In such a way, the explanation of the weights and consequently dispatch results to the participants could be easier.

It is proven that for any positive weights chosen, the optimal solution of the problem (3.2) is a Pareto-optimal solution of (2.3). However, only in the case when (2.3) is an LP, it is possible to choose weights in a way to generate any possible Pareto-optimal solution. This is not generally true for other classes of multi-criteria problems [68]. However, this is not critical in our application, as in the SGI dispatch problem we want to generate just one efficient solution fast. It is not required to generate the complete Pareto front, since it would be of little help in the stressful dispatcher's decision process, where decisions are made under high pressure.

3.4.3 Maxi-min aggregation

In this Section we cite the mathematical formulation after Ogryczak [68]. The set of solutions of the maxi-min aggregation, formulated in (3.3) contains an effective solution of (2.3) and the unambiguous solution of (3.3) is an efficient solution of (2.3). In the case that there exist multiple optimal solutions to (3.3), only one will be an efficient solution of (2.3).

$$
\begin{aligned}
\max \quad & z \\
\text{s.t.} \quad & z \leq f_k(\boldsymbol{x}) \qquad \forall k = 1, \ldots, p, \\
& \boldsymbol{x} \in Q
\end{aligned}
\tag{3.3}
$$

The drawback of possibly not generating an efficient solution may be corrected by adding a small regularising component making (3.3) equivalent to two-stage lexicographic maximisation. Such a reformulation is shown in (3.4).

$$
\begin{aligned}
\max \quad & z + \rho \sum_{i=k}^{p} f_k(\boldsymbol{x}) \\
\text{s.t.} \quad & z \leq f_k(\boldsymbol{x}) \qquad \forall k = 1, \ldots, p, \\
& \boldsymbol{x} \in Q
\end{aligned}
\tag{3.4}
$$

where: $0 < \rho \ll 1$ — small positive regularising parameter.

Discussion: This aggregation represents the strategy, where we want to maximise the lowest outcome. When referring to the SGI dispatching, it would represent maximising the smallest outcome of the least favoured participant. The results of this strategy can be easily explained to the participants involved. However, it only looks at the worst outcome and the averaged value of all outcomes (in case of formulation (3.4)) not distinguishing between quality and cost-based ones. The aggregation does not introduce any means to represent participants' preferences for the optimisation. The aggregation is introduced by means of p additional linear constraints. Although they increase the computational complexity, the increase should not significant due to their linearity.

3.4.4 Lexicographic maximisation

In this Section we cite the mathematical formulation after Ogryczak [68]. This technique is formulated in (3.5). Lexicographic maximisation optimises the criteria in the dictionary order. If the lexicographic scalarisation defines a preference relation, that is strictly monotonic, then the optimal solution to the lexicographic maximisation problem is an efficient solution of (2.3). Although not directly visible from the formulation, the technique is still considered an aggregation [68].

$$
\begin{aligned}
\text{lex max} \quad & [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_p(\boldsymbol{x})] \\
\text{s.t.} \quad & \boldsymbol{x} \in Q
\end{aligned}
\tag{3.5}
$$

The lexicographic maximisation can be exactly implemented by sequentially optimising p times each of the criteria. In each problem, an additional equality constraint on already optimised criteria is added. This is shown in Algorithm 1, where variable $\boldsymbol{Optsol}$ stores the trade-off solution of the final problem and $\boldsymbol{Optval}$ stores the criteria value of this solution and $\boldsymbol{x}^*$ is the currently obtained optimal solution.

Algorithm 1 Exact lexicographic maximisation

$\boldsymbol{d} \leftarrow \emptyset$

Solve $\{\max f_1(\boldsymbol{x}) : \boldsymbol{x} \in Q\}$

$d_1 \leftarrow f_1(\boldsymbol{x}^*)$

$k \leftarrow 2$

while $k \leq p$ **do**

  Solve $\{\max f_k(\boldsymbol{x}) : f_i(\boldsymbol{x}) = d_i \; \forall i = 1, \ldots, k-1; \; \boldsymbol{x} \in Q\}$

  $d_k \leftarrow f_k(\boldsymbol{x}^*)$

  $k = k + 1$

end while

$\boldsymbol{Optsol} \leftarrow \boldsymbol{x}^*$

$\boldsymbol{Optval} \leftarrow \boldsymbol{f}(\boldsymbol{x}^*)$

The algorithm allows for the computation of exact lexicographic solutions. However, it significantly increases the computational burden, as the problem considered must be solved p times.

This drawback can be overcome if a lexicographic approximation is used instead of the exact approach. For this, one may leverage the weighted sum aggregation (3.2) with positive and strictly decreasing weights. Such an approach is then called *Ordered Weighted Aggregation* (OWA) [132]. The weighted sum approach approximates the lexicographic approximation when the differences between weights tend to infinity. This is shown in (3.6).

$$\begin{aligned} \max \quad & f(\boldsymbol{x}) = \sum_{k=1}^{p} w_k \, f_k(\boldsymbol{x}) \\ \text{s.t.} \quad & \boldsymbol{x} \in Q \end{aligned} \tag{3.6}$$

where $w_1 \gg w_2 \gg w_3 \gg \ldots \gg w_p > 0$.

Discussion: The lexicographic maximisation gives some extensive flexibility in crafting the optimisation problems. However, it will not be tangible when directly applied to (2.3), as the question will arise on how to define the dictionary order. When defined by the dispatcher, it may not be easily understandable by the SGI participants.

Although not directly applicable to (2.3), OWA may be a powerful tool to solve (2.5) to find an equitably efficient solution of the general SGI dispatch optimisation problem. Lexicographic maximisation will then aim at improving outcomes for all participants, starting from the least favoured one. However, the approach does not stop there but continues to improve all outcomes. Such behaviour will be easier to be explained.

3.4.5 Reference Point Method

In this Section we describe the method developed by Wierzbicki [133, 134, 111]. Reference Point Method (RPM) is an interesting technique, which leverages specially crafted scalarising functions, called *achievement functions*. Values of these functions can be simply understood as the numerical representation of the Decision Maker's satisfaction on achieving a given value of the precise criterion. They intend to make the criteria comparable between each other by leveraging the vectors of so-called *aspirations* $\boldsymbol{a}$ and *reservations* $\boldsymbol{r}$. Aspirations give the desired/best values of the criteria to take, and reservations give the still acceptable values of the criteria (yet not best), $r_k < a_k \; \forall k = 1, 2, \ldots, p$. There exists a version of RPM which considers only the aspiration vector $\boldsymbol{a}$ (without $\boldsymbol{r}$) to calculate the trade-off solution, however, since it is less flexible than the one considering also reservations, we do not cite it in this Section. What is more, Kruś et al. have proven the formulation with reservations valuable to help in the market decision problems with multiple players [135, 136].

RPM technique builds on the claim that a solution with all criteria that meet the aspirations is preferred to the one giving at least one criterion not meeting the associated aspiration. Similarly, a solution giving all criteria that meet the reservations associated with them is preferred to the one in which at least one criterion does not reach its reservation.

The achievement function for a given criterion k is given in (3.7). We cite here the formulation proposed in [137]

$$s_k(f_k(\boldsymbol{x}), a_k, r_k) = \begin{cases} \gamma \frac{f_k(\boldsymbol{x}) - r_k}{a_k - r_k} & \text{for} \quad f_k \leq r_k, \\ \frac{f_k(\boldsymbol{x}) - r_k}{a_k - r_k} & \text{for} \quad r_k < f_k \leq a_k, \\ \beta \frac{f_k(\boldsymbol{x}) - a_k}{a_k - r_k} + 1 & \text{for} \quad a_k < f_k, \end{cases} \tag{3.7}$$

where β, γ are arbitrarily chosen parameters, such that $0 < \beta < 1 < \gamma$.

Under the above assumption, achievement functions s_k are strictly increasing. Function s_k increases sharply until the point where $f_k(\boldsymbol{x})$ reaches r_k. In the point where $f_k(\boldsymbol{x}) = r_k$, $s_k = 0$. After passing r_k, the slope of increase is less steep, up to the point where $f_k(\boldsymbol{x}) = a_k$. In this point $s_k = 1$. When $f_k(\boldsymbol{x}) > 1$, an increase of s_k is still possible, yet the slope is very gentle. This behaviour in the problem of maximisation is shown in Fig. 3.1.

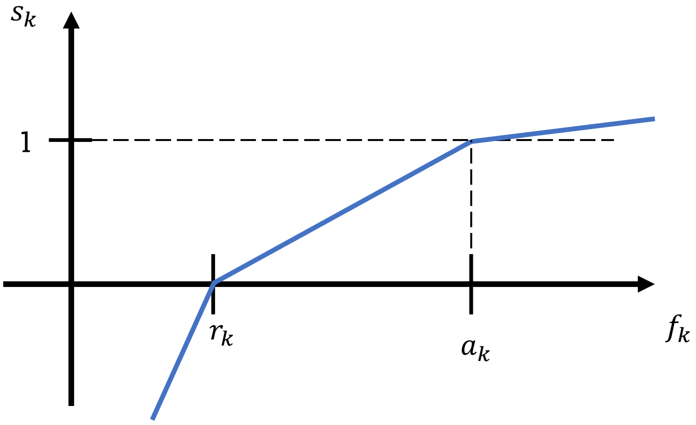


Figure 3.1: Achievement function of f_k (own elaboration basing on [137]).

In its original formulation, RPM leverages the maxi-min aggregation of achievement functions, with a small regularising component to ensure the Pareto-optimality of the solution as in (3.4). In other words, it aims to maximise the smallest value of the achievement functions and as such, to maximise DM's satisfaction with the worst outcome. Having said the above, one may formulate the RPM aggregation problem as (3.8).

$$
\begin{aligned}
\max \quad & v + \rho \sum_{k=1}^{p} z_k \\
\text{s.t.} \quad & v \leq z_k \qquad \forall k = 1, \ldots, p, \\
& z_k \leq \gamma \frac{f_k(\boldsymbol{x}) - r_k}{a_k - r_k} \quad \forall k = 1, 2, \ldots, p, \\
& z_k \leq \frac{f_k(\boldsymbol{x}) - r_k}{a_k - r_k} \quad \forall k = 1, 2, \ldots, p, \\
& z_k \leq \beta \frac{f_k(\boldsymbol{x}) - a_k}{a_k - r_k} + 1 \quad \forall k = 1, 2, \ldots, p, \\
& \boldsymbol{x} \in Q
\end{aligned}
\tag{3.8}
$$

Discussion: The concept of the achievement functions, which take into consideration aspirations and reservations as participants' preferences for oprimisation, is appealing. In real-life SGI dispatch optimisation problem, those values would be provided by dispatching participants (as proposed in [13]) or would be based on expert guidelines. An example of the latter is the estimation of aspirations/reservations for the assignment of patients to the Emergency Departments using guidelines like [138]. In that sense, these values would be easily explainable to the dispatch participants and would allow for direct reflection of their needs towards the criteria.

The above-mentioned RPM aggregation method, when applied to the original problem (2.3), does not guarantee the equitable efficiency of the solutions obtained. However, there is nothing holding us back from applying any one of them to the re-formulated problem (2.5), which would guarantee the equitable efficiency of the solution. The only change one should make is to understand $\boldsymbol{f}(\boldsymbol{x})$ shown in (3.2) - (3.8) as $\boldsymbol{z}$ in (2.5). One can also solve (2.5) in combination with achievement functions as in (3.8), together with a different goal function than the classic maxi-min one (as in RPM). An example of such a re-formulation using OWA and RPM's achievement functions is given in (3.9). It is called Nucleolar RPM [139]. Nucleolar RPM gives advantages of both ease of understanding and explaining parameterisation of the Pareto-front, which is provided by the achievement functions, and the proven generation of efficient solutions. What is more, thanks to the use of OWA, we are sure that the criteria are improved through optimisation, in the order from the least favoured criterion, to the most favoured one.

$$
\begin{aligned}
\max \quad & o(\boldsymbol{x}) = \sum_{k=1}^{p} w_k g_k(\boldsymbol{x}) \\
\text{s.t.} \quad & g_k = k t_k - \sum_{i=1}^{p} d_{i,k} \quad k = 1, 2, \ldots, p, \\
& t_k - d_{i,k} \leq z_i(\boldsymbol{x}) \qquad \forall k, \forall i = 1, ..., p, \\
& z_k \leq \gamma \frac{f_k(\boldsymbol{x}) - r_k}{a_k - r_k} \quad \forall k = 1, 2, \ldots, p, \\
& z_k \leq \frac{f_k(\boldsymbol{x}) - r_k}{a_k - r_k} \quad \forall k = 1, 2, \ldots, p, \\
& z_k \leq \beta \frac{f_k(\boldsymbol{x}) - a_k}{a_k - r_k} + 1 \quad \forall k = 1, 2, \ldots, p, \\
& \boldsymbol{x} \in Q
\end{aligned}
\tag{3.9}
$$

where $w_1 \gg w_2 \gg w_3 \gg \ldots \gg w_p > 0$.

3.4.6 Goal Programming

Goal Programming (GP) is a scalarisation technique which also leverages the concept of aspirations — like RPM. The approach aims to minimise the deviations (both positive and negative) between the criterion value and the aspiration associated with it. A number of traditional GP formulations have been developed, namely: weighted sum, mini-max, and lexicographic GP approach [140]. A simple version of the weighted sum GP is presented in (3.10).

$$
\begin{aligned}
\min \quad & e(\boldsymbol{x}) = \sum_{k=1}^{p} (w_k^+ e_k^+ + w_k^- e_k^-) \\
\text{s.t.} \quad & f_k(\boldsymbol{x}) + e_k^+ - e_k^- = a_k \qquad \forall k = 1, 2, \ldots, p, \\
& e_k^+, e_k^- \geq 0 \qquad \forall k = 1, 2, \ldots, p, \\
& \boldsymbol{x} \in Q
\end{aligned}
\tag{3.10}
$$

where e_k^+, e_k^- represent the negative and positive deviation of $f_k(\boldsymbol{x})$ from a_k respectively. Additionally, condition $e_k^+ e_k^- = 0$ must also hold, yet this is implicitly satisfied through the nature of GP minimisation [141].

Unfortunately, the original formulation of GP does not guarantee the Pareto-optimality of the solution. It only suggests decisions which result in outcomes being closest to pre-defined aspirations [142]. However, there exist some algorithms which can detect the Pareto inefficiency of solutions and some formulations which allow for Pareto-optimality restoration [141, 142]. Due to the argumentation given in the Discussion part of this Section, they are not cited here.

Discussion: Goal Programming might be a tempting technique to be applied in the SGI dispatch optimisation problem. This is mainly because it is possible to directly express participants' needs towards criteria by means of aspirations. This would be easily explainable to the participants. What is more, despite the fact that GP in its original formulation does not guarantee the Pareto-optimality of the solution, some techniques exist which can allow for the restoration of Pareto - optimality.

However, one can easily remark that the standard GP formulation as given in (3.10) may not be directly and easily implementable to the equitable problem (2.5). This is because, in the straightforward implementation, one should identify aspirations for $\overline{\Theta}_k \ \forall k = 1, 2, \ldots, p$. Obviously, this is of little practicality when applied to the SGI dispatch optimisation problem since these values will most likely be impossible to be accurately estimated. As argued by Ogryczak [142], the Reference Point Method is an enhancement of the GP. Therefore, we propose to use RPM instead of GP, as it may be more easily applicable/implementable with respect to (2.5), and it follows similar principles as GP.

3.4.7 ε-constraint method

Here we cite the mathematical formulation after Haimes [143]. This method transforms (2.3) in a way that a single criterion $f_h(\boldsymbol{x}),\ h \in \{1, 2, \ldots, p\}$ is optimised when constraints on other criteria are imposed. It can be proven that this method always yields weakly efficient solutions. This technique is formulated in (3.11).

$$\begin{aligned} \max \quad & f_h(\boldsymbol{x}) \\ \text{s.t.} \quad & \varepsilon_k \leq f_k(\boldsymbol{x}) \quad \forall k \in \{1, \ldots, p\} \backslash \{h\}, \\ & \boldsymbol{x} \in \mathcal{Q} \end{aligned} \tag{3.11}$$

where ε_k is an arbitrarily chosen constraint imposed on $f_k(\boldsymbol{x})$.

Discussion: The ε-constraint method gives an easy possibility to generate the complete Pareto front. This can be accomplished by changing the elements of $\boldsymbol{\varepsilon}$ with a small step. This property is, however, not of utmost importance in the SGI dispatch optimisation problem — see

Discussion in Sec. 3.4.2.

This method may be of little applicability to the SGI dispatching since one should identify the lower bound on all criteria but the one being optimised. Unfortunately, this would likely not be possible since the minimum values of criteria will not be known at the moment of dispatching. What is more, even if they were known to the dispatcher, it would mean that only one criterion associated with one participant would be maximised and the others would simply be fixed under the *not worse than* principle. This would likely not be easily explainable to the participants concerned. It would be difficult to say why a particular participant was chosen to have its service maximised. Yet, the method could potentially allow for consideration of participants' requirements on criteria if they are transformed into the form of lower bound constraints. However, allowing the participants to assign their lower bounds themselves could lead to infeasibility of the problem.

3.4.8 Metaheuristics

There exists a research trend which investigates the use of different metaheuristics to solve multi-criteria optimisation problems. Some of them are: evolutionary algorithms, simulated annealing, tabu search or others [144]. They can quickly yield valuable solutions to both linear and nonlinear problems, yet some of them might not guarantee the Pareto-optimality of the results. In this work, we focus purely on mathematical programming methods and, as such, consider the metaheuristics as out of scope. What is more, due to the fact that they form a completely different class of problems than the aggregation methods described, we do not compare them in the table at the end of this Section.

3.4.9 Theoretical comparison between solution strategies

In this Section, we provide a summary of the Discussion points provided in previous Sections, which is based on a theoretical overview of the methods presented in Sec. 3.4.2 - 3.4.7. We take the following characteristics for comparison.

1. Guarantee of Pareto-optimality of the solution;
2. Understandability/ease of assignment of parameters/results, when applied to (2.3);
3. Understandability/ease of assignment of parameters/results, when applied to (2.5);
4. One-step implementation (meaning that there is no need to write a dedicated algorithm for the problem);
5. Possibility of directly considering the participants' requirements for the values of criteria.

The summary is given in Tab. 3.1, where "+" means "true", "–" means "false", "+/–" represents "partially true", "+d" — "generally true but depends on formulation", "–d" — "generally false but exist formulations which can mitigate the problem". Techniques compared are shown in columns and are labelled with the number of Sections in which they were discussed.

Table 3.1: Theoretical comparison of different multi-criteria solution techniques

Properties of multi-criteria solution techniques						
Characteristic no.	3.4.2	3.4.3	3.4.4	3.4.5	3.4.6	3.4.7
1	+	+d	+	+	–d	+
2	–	+	–	+	+	–
3	+	+	+	+	–	–
4	+	+	+d	+	+	+
5	–	–	–	+	+	+/–

From the Tab. 3.1, one can conclude that, theoretically, the most suitable solving technique for the multi-criteria generic SGI dispatch optimisation problem with quality-based criteria is the Reference Point Method (presented in Sec. 3.4.5) as it scored highest in the comparison table presented. Thus, in the next Chapters, we select the RPM technique as the solution strategy applied.

3.5 Summary

In this Chapter the generic SGI dispatch optimisation problem with quality-based criteria was formulated and discussed from the functional perspective. The proposed problem allows for its more specific formulations to be used to help with dispatching in multiple various classes of SGIs. In the course of the functional theoretical analysis it has been shown that the proposed problem formulation may respond well to the design requirements given in Sec. 1.4.

The proposed optimisation problem is a multi-criteria one. Thus, to solve it to Pareto-optimality using the mathematical optimisation methods, it is needed to use one of the scalarisation techniques. We have theoretically reviewed, compared and discussed some of the well-known techniques with respect to the problem presented. In the comparison looked at five characteristics — guarantee of Pareto-optimality of the result and four other characteristics related to the appropriateness to the SGI dispatch. From our investigations it stands out that the Reference Point Method scalarisation appears to be best suited for the multi-criteria dispatch optimisation of the SGIs as it scored highest in the comparison. Therefore, in the remainder of this work it is the technique used for all numerical tests and comparisons.

Chapter 4

Considering multiple criteria in the generic SGI dispatch optimisation problem — experiments

This Chapter gives the results of numerical experiments of applying the generic SGI dispatch optimisation problem with quality-based criteria to synthetic test Cases. We investigate three Cases — A, B and C, each one with a differing number of customers. Each Case consists of 2,000 test instances, giving in total 6,000 experiments. Each one of them is solved using three different formulations and compared statistically over some defined indices.

This Chapter is structured as follows: first, we formulate the specific problem which is used for the investigations. Then, we describe the materials and methods used for the tests including the indices considered and the statistical test used. After this, we present and analyse the results for each of the Cases separately, every time summarising the results obtained. The Chapter is finished by presenting the conclusions and discussing the results.

4.1 Introduction

In this Chapter, we intend to investigate some basic properties of the generic SGI dispatch optimisation problem with quality-based criteria SDOQ (3.1). We also intend to verify experimentally the main research thesis of this work, namely whether ***adding quality-based criteria to the generic SGI dispatch optimisation problem adds value to the dispatching, allowing to yield more tailored results to the customers' needs.***

For this purpose, we test the approach on a test Case given in (4.1), which is a specific realisation of the multi-period SDOQ problem (3.1). It represents a generic single-period **S**GI **A**ssignment **O**ptimisation problem with **Q**uality-based criteria (**SAOQ**). It can, for instance, reflect an assignment of ambulances (members of the set $\mathcal{P}_s$) to patients in need of urgent care (members of the set $\mathcal{P}_k$). After proper scaling, each ambulance can be described on the speciality it offers towards dealing with a given health condition (quality part) and on the time it takes for the ambulance to reach a given patient (cost part). Participants are then understood as patients seeking emergent care, and hence, the quality criteria represent the speciality received by patients and the cost criteria — time of waiting for the care to arrive on the scene. Therefore, we omit the index m from the SDOQ problem (3.1) and use only the index of customers (i) with enumeration of different criteria n, k. As the number of time instances (reflected in the SDOQ problem (3.1) as T) equals 1, the assignment variables are two-dimensional and not three-dimensional as in the SDOQ problem (3.1). The demand Δ_i is equal to 1 as exactly one SGI resource is to be assigned to each participant. The set $\mathcal{Q}$ in this example is not specified, as all constraints appear directly in the formulation of SDOQ problem (3.1).

SAOQ:

$$
\begin{aligned}
\max \quad & [q_{1,1}, -c_{1,1}, q_{1,2}, -c_{1,2}, \ldots, q_{\bar{i},\bar{n}}, -c_{\bar{i},\bar{k}}] \quad \bar{i} = |\mathcal{P}_k|, \bar{n} = |\mathcal{I}|, \bar{k} = |\mathcal{K}| \\
\text{s.t.} \quad & q_{i,n} = f^1_{i,n}(\boldsymbol{x}) \qquad \forall i = 1, \ldots, \bar{i} \quad \forall n = 1, \ldots, \bar{n}, \\
& c_{i,k} = f^2_{i,k}(\boldsymbol{x}) \qquad \forall i = 1, \ldots, \bar{i} \quad \forall k = 1, \ldots, \bar{k}, \\
& f^1_{i,n}(\boldsymbol{x}) = \sum_{j=1}^{|\mathcal{P}_s|} y^j_i \; d^j_{i,n} \qquad \forall i = 1, \ldots, \bar{i} \quad \forall n = 1, \ldots, \bar{n}, \\
& f^2_{i,n}(\boldsymbol{x}) = \sum_{j=1}^{|\mathcal{P}_s|} y^j_i \; t^j_{i,k} \qquad \forall i = 1, \ldots, \bar{i} \quad \forall k = 1, \ldots, \bar{k}, \\
& \sum_{j=1}^{|\mathcal{P}_s|} y^j_i = 1 \qquad \forall i = 1, \ldots, \bar{i}, \\
& y^j_i \in \{0, 1\} \qquad \forall i = 1, \ldots, \bar{i} \quad \forall j = 1, \ldots, |\mathcal{P}_s|
\end{aligned}
\tag{4.1}
$$

where:

- y^j_i — assignment binary variable of supplier j to customer i. Takes 1, when j is assigned to to i and 0 otherwise;
- $d^j_{i,n}$ — parameter describing what value for quality criterion n, supplier j can offer to customer i;
- $t^j_{i,k}$ — parameter describing what value for cost criterion k, supplier j can offer to customer i.

Basing on the theoretical comparison of the aggregation methods presented in Sec. 3.4, we have concluded that the Reference Point Method (RPM) appears to be the most appropriate method for solving the problem considered. Therefore, for experiments presented in this Chapter, we use the RPM aggregation whenever solving any multi-criteria problems.

4.2 Materials and methods

The tests are performed for the SAOQ problem, under three Cases — A, B and C. In order to verify the impact of the formulation on obtained dispatching result in every one of them, we consider and compare three variations of the problem (4.1), namely

- Single-criterion problem, where the scalar objective is to minimise the sum of cost criteria (B.1);
- Non-equitable multi-criteria problem (B.2);
- Equitable multi-criteria problem (B.3).

Formulations of each of the three variations are given in Appendix B, and the parameters assumed were: $\gamma = 10^3, \beta = 10^{-3}, \rho = \frac{10^{-4}}{n_c}$, where n_c is the total number of the criteria considered. The problems were coded with Matlab with CVX modelling package [145, 146] installed and solved with Gurobi on a laptop equipped with the Intel Core i5-4210U processor.

For test purposes, we randomly generate the values of aspirations, reservations and the values of $d^j_{i,n}, t^j_{i,k}$ from the uniform distribution [0,1], i.e., $\boldsymbol{a}, \boldsymbol{r}, \boldsymbol{d}, \boldsymbol{t} \sim \mathcal{U}(0,1)$, forming synthetic test instances. For each Case, 2,000 instances are randomly generated and solved, with different values of parameters. In total, this gives 6,000 numerical experiments performed. Then, the results are compared statistically between each other.

We analyse the three test Cases as they differ importantly in size of the problems. The size increases in the dictionary order — Case A being the smallest and Case C the largest one. In such way we can investigate the results for the generic problem regardless of its complexity. We have already stated that the formulation (4.1) may represent for example, the task of dispatching ambulances to patients taking into consideration both the time-to-arrival and the quality of the EMS service criteria. However, due to the random nature of all of the 6,000 test instances and the further statistical inference applied, the conclusions drawn may be generalised to other SGI dispatch problems.

4.2.1 Indices under consideration

We take a *functional* approach to evaluate the dispatching results. By *functional*, we mean that we compare some performance measures (indices) directly related to the dispatch results. We assume here that the values of aspirations and reservations reflect well their preferences towards the criteria. In the example of ambulance dispatching, the aspirations/reservations would be estimated basing on the patient's medical condition, following medical guidelines (e.g., [138]) and therefore assumed as well-reflecting the desires. It is worth reminding here that the value of reservation still reflects an *acceptable* value of the criterion associated.

Having said the above, we give in this Section the test indices, values of which will be subject to comparison, to answer the main research question. They are directly linked to the value of $c_{i,k}$ and $q_{i,n}$ as obtained through the dispatch optimisation process. The performance measures (indices) considered are shown in the bullet points below:

- Number of criteria being at least as good as their reservations associated;
- Number of criteria not meeting their reservations by at least 10%;
- Maximum strictly positive percentage gap between the criterion value and its reservation, for both cost and quality criteria jointly;
- Mean value of the utility function;
- Optimiser's solution time.

.

4.2.2 Statistical analyses

In order to draw conclusions regardless of possible random character of the results obtained, we compare them statistically. Firstly, descriptive statistics drawn from the samples are presented and shown in box plots for visual comparisons. In some cases, there exist outliers in data, which make the box plot hard to read and interpret. Hence, to draw the box plots, outliers are detected and replaced with the use of Matlab's $filloutliers$ function. Parameters are set to identify as outliers the points located more than three scaled median absolute deviations (MAD) from the median and to fill them using the Modified Akima piecewise cubic Hermite interpolation. The exact definition of those parameters and the description of the function itself can be found in the Matlab documentation [147]. By default, the filling of outliers is applied only to the drawing of the box plots and the tables report data without filling them, unless clearly specified in the appropriate Sections of the text. Similarly, test statistics are calculated on data without the outliers filled, unless it is clearly stated in the text.

Secondly, a statistical inference test for the comparison of two mean values is applied to see if any pair of them is significantly different at a given significance level α. We always give in the respective Sections the test statistic calculated.

The statistical test under consideration is analysed in [148] and is cited in this Section. This test has been successfully used by Drabecki and Kułak to identify energy market behaviours during the COVID-19 pandemic times [149]. No assumption is made on the distributions of random variables considered, provided that a sufficiently large number of observations are collected. Specifically, no assumption is made on the normality of these distributions. This property makes the test a powerful tool for the experiments since the samples used are not drawn from the normal distribution, which is expected by the nature of parameter generation and has also been verified through the one-sample Kolmogorov-Smirnov test [150].

Statistical Test: Let X and Y be independent random variables of any possible unknown distributions and unknown finite variances. Specifically, no assumption on the normality of the distribution is made. Let also μ_X, μ_Y be their mean values. Provided that independent random samples x and y of X and Y respectively were collected, with $n_X \geq 100, n_Y \geq 100$ observations of each variable, we can test at the significance level α a null hypothesis:

H_0**:** $\mu_X = \mu_Y$

against the alternative hypothesis H_1

H_1**:** $\mu_X > \mu_Y$

The test statistic is formulated as (4.2)

$$W = \frac{\overline{x} - \overline{y}}{\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}} \tag{4.2}$$

where

- $\overline{x}, \overline{y}$ — mean values calculated on samples x and y respectively;
- s_X^2, s_Y^2 — sample variances of x and y respectively.

The rejection region of the null hypothesis is given in (4.3)

$$Z = (z_{1-\alpha}, \infty) \tag{4.3}$$

where $z_{1-\alpha}$ — quantile of order $(1 - \alpha)$ of the normal distribution, with mean value equal to zero and variance to one, referred to as $\mathcal{N}(0, 1)$, which for $\alpha = 0.05$ equals 1.6449.

4.3 Results — Case A

This Section presents the test setup summary of this Case presented in bullet points below. Due to space limitations, we do not cite here the exact values of the random parameters chosen. However, they are made available in the online repository, under the link: `https://bit.ly/3rPZATG`.

Test setup:

- Number of customers $|\mathcal{P}_k| = 10$;
- Number of suppliers/resources $|\mathcal{P}_s| = 13$;
- Number of criteria for each $i \in \mathcal{P}_k = 2$ (one cost criterion, one quality criterion), i.e., total number of criteria equals 20;

- Randomly generated aspirations, reservations, $d^j_{i,n}$ and $t^j_{i,k}$;
- In total, 2,000 instances were generated, solved and compared.

As described in Sec. 4.2, we solve each of the test instances three times. Each time by means of a different optimisation problem, namely (B.1) - (B.3), which obviously results in three different Pareto-optimal solutions for each instance. This Section describes how these solutions impact the indices considered in Sec.4.2.1.

4.3.1 Number of criteria being at least as good as reservations

This index is presented jointly for both cost and quality criteria. The logic behind such a comparison is that in a real-life dispatching problem, the dispatcher should assure that as many outcomes as possible fall within their target interval of values. No matter if this is for cost or for quality. A solution should be considered more tailored to needs if it is linked with a higher number of outcomes meeting their target intervals. Such a test approach shows a high-level *master* comparison.

The summary of experimental results for Case A, performed to investigate the total number of criteria, which are as good as their reservation, is given in Tab. 4.1 and in Fig. 4.1. One can immediately notice that the multi-criteria problems allow for giving more tailored results in terms of the number of criteria being at least as good as their reservation.

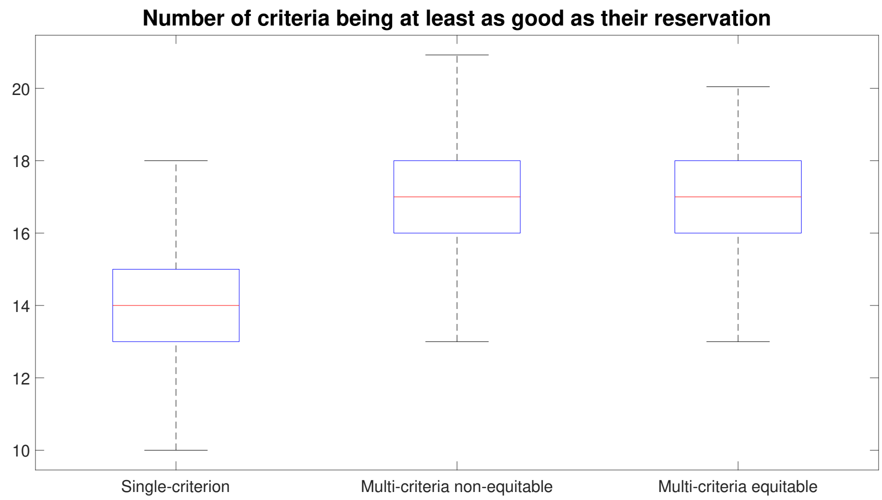


Figure 4.1: Number of criteria at least as good as their reservations (Case A)

Table 4.1: Number of criteria being at least as good as reservations (Case A)

Number of criteria being at least as good as reservations Case A			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	20.00	20.00	20.00
Min value	8.00	7.00	0.00
Mean value	14.29	16.48	17.02
Median value	14.00	17.00	17.00
Mode	14.00	17.00	17.00
Standard deviation	1.79	2.12	1.79
IQR	2.00	3.00	2.00

Conclusions drawn from visually comparing the data are also confirmed statistically. As null hypotheses, we say that: *The mean values of the number of criteria being at least as good as the reservation levels are equal for any two problems considered* and the alternative hypotheses are stated in points below. These points specify also the resulting test statistics (4.2).

1. The mean value of the number of criteria being at least as good as the reservation level in (B.2) is greater than (B.1), $W = 35.40$;
2. The mean value of the number of criteria being at least as good as the reservation level in (B.3) is greater than (B.1), $W = 48.35$;
3. The mean value of the number of criteria being at least as good as the reservation level in (B.3) is greater than (B.2), $W = 8.68$.

Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means and accept all of the above alternative ones.

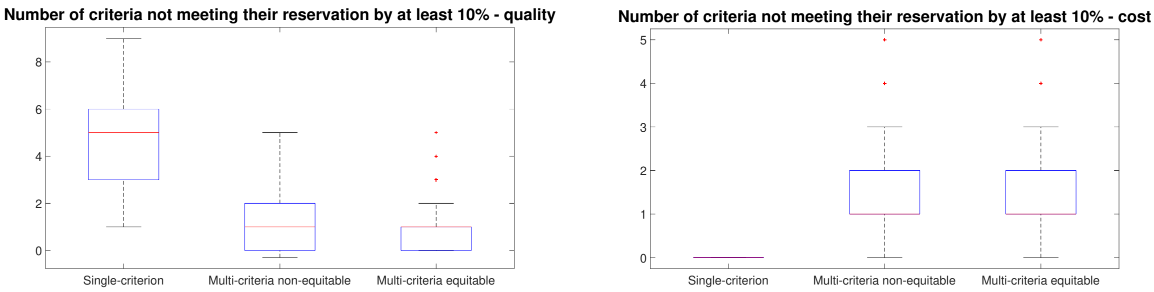
In conclusion, the multi-criteria problems results allowed for producing results with a significantly higher number of criteria reaching or outperforming their reservation. What is more, the test has shown that the multi-criteria equitable problem also allowed for reaching a significantly higher number of reservations than the non-equitable multi-criteria one.

4.3.2 Number of criteria not meeting their reservations by at least 10%

In this Section, we investigate the number of criteria, which are worse by at least 10% than their respective reservations. To assure more clarity and visibility of the results, this index is presented separately for the cost criterion and for the quality criterion. The reason it is separated is that this time, we want to dive deep into the results produced. We want to investigate if there are differences between comparing cost criteria and the quality ones. This is in contrast with the previously studied index in Sec. 4.3.1, where we show a high-level comparison.

The summary of results obtained is shown in Tab. 4.2 and in Tab. 4.3, as well as in Fig. 4.2. One can see that when it comes to meeting the reservations for the quality criteria, the multi-criteria approaches perform significantly better. Differences in mean, median and mode values between the single-criterion and multi-criteria approaches are seen. What is more, the equitable approach performs better than the non-equitable one. This is according to expectations as in the multi-criteria approaches the quality criteria are controlled, which is not the case in the single-criterion one. In addition to that, it is also expected that the equitable approach would generally perform better than the non-equitable one, as it follows the principle of transfers.

However, this is not the case for the cost criteria. In this case, the single-criterion performs better than the other ones considered. This is also according to expectations, as the single-criterion approach yields the optimal result for cost. However, it is worth noting that the difference between (B.1) and (B.2)-(B.2) when considering the quality criteria is much higher than when considering the cost ones. This might be also observed in the statistical analyses, as the test statistics differ a lot.



(a) quality

(b) cost

Figure 4.2: Number of criteria not meeting their reservations by at least 10% (Case A)

Table 4.2: Number of quality criteria not meeting their reservations by at least 10% (Case A)

Number of quality criteria not meeting their reservations by at least 10% Case A			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	9.00	7.00	5.00
Min value	0.00	0.00	0.00
Mean value	4.51	1.05	0.73
Median value	5.00	1.00	1.00
Mode	5.00	0.00	0.00
Standard deviation	1.59	1.13	0.82
IQR	3.00	2.00	1.00

Table 4.3: Number of cost criteria not meeting their reservations by at least 10% (Case A)

Number of cost criteria not meeting their reservations by at least 10% Case A			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	5.00	7.00	5.00
Min value	0.00	0.00	0.00
Mean value	0.64	1.47	1.27
Median value	0.00	1.00	1.00
Mode	0.00	1.00	1.00
Standard deviation	0.76	1.14	1.02
IQR	1.00	1.00	1.00

Similarly to the previous experiment, mean values of the index taken in three groups, over the whole set of 2,000 instances are compared statistically. The alternative hypotheses together with the respective test statistics are shown in the following bullet points. Firstly, we give the comparisons for the null hypothesis stated as: *The mean values of number of quality criteria not meeting their reservations by at least 10% are equal for any two of the problems considered*. The alternative hypotheses, with the resulting test statistics, are given in the points below.

1. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.1) is greater than in (B.2), $W = 79.23$;

2. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.1) is greater than in (B.3), $W = 93.98$;

3. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.2) is greater than in (B.3), $W = 10.06$.

Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means of the number of quality criteria being worse than their reservations by at least 10%, and accept all of the above alternative hypotheses.

Secondly, we give the comparisons for the null hypothesis stated as: *The mean values of number of cost criteria not meeting their reservations by at least 10% are equal for any two of the problems considered*. The alternative hypotheses, with the resulting test statistics, are given in the points below.

1. Mean value of the number of cost criteria not meeting their reservations by at least 10% in (B.2) is greater than in (B.1), $W = 27.08$;

2. Mean value of the number of cost criteria not meeting their reservations by at least 10% in (B.3) is greater than in (B.1), $W = 22.17$;

3. Mean value of the number of cost criteria not meeting their reservations by at least 10% in (B.2) is greater than in (B.3), $W = 5.78$.

Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means of the number of cost criteria being worse than their reservations by at least 10%, and accept all of the above alternative hypotheses.

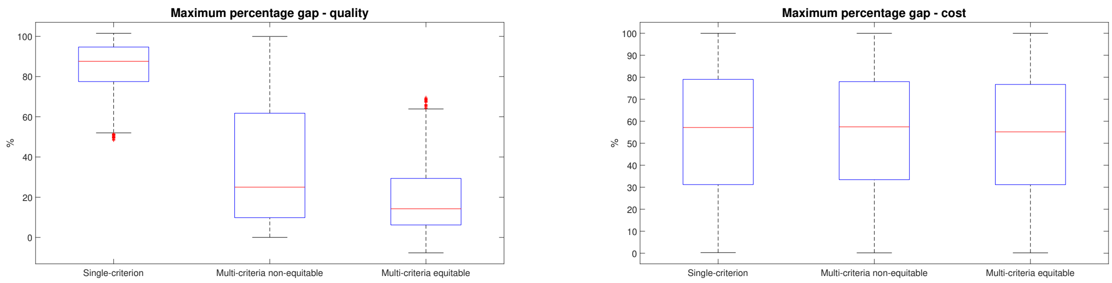
As already stated, thanks to the above statistical tests, we can state that the multi-criteria approaches perform better than the single-criterion approach for the quality criteria and vice versa for the cost criteria. However, when we look at the test statistics, we may conclude that the loss of performance for cost criteria is not as large, as the gain in performance for quality criteria. What is more, the equitable approach allowed to produce significantly lower mean numbers of criteria values, which were worse by at least 10% than their reservation for the non-equitable one.

4.3.3 Maximum percentage gap

Similarly to the previous Section, we consider this index as a deep dive into the results obtained. This is why we investigate it separately for quality and for cost criteria. The percentage gap is calculated as

$$g^i_{\%} = \begin{cases} 100\% \frac{f_i(\boldsymbol{x}) - r_i}{f_i} & \text{for criteria being minimised,} \\ 100\% \frac{r_i - f_i(\boldsymbol{x})}{r_i} & \text{for criteria being maximised} \end{cases} \tag{4.4}$$

where $f_i(\boldsymbol{x})$ is the value of i^{th} criterion. In the experiments, we take the maximum value of the percentage gap over all quality/cost criteria in a given test instance $mg = \max_i g^i_{\%}$. Since it is also meant to give more insights into the performance of the methods, we study the quality and cost criteria separately. What is more, we take into consideration only strictly positive gaps. A summary of the results obtained is presented in Fig. 4.3 as well as in Tab. 4.4 and Tab. 4.5.



(a) quality (b) cost

Figure 4.3: Maximum percentage gap (Case A)

Table 4.4: Maximum percentage gap — quality (Case A)

Maximum percentage gap — quality [%] Case A			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	100.00	99.89	99.89
Min value	7.43	0.00	0.00
Mean value	82.20	36.08	26.74
Median value	86.36	24.65	17.24
Mode	7.43	0.00	0.00
Standard deviation	15.85	30.96	25.58
IQR	20.22	52.43	33.83

Table 4.5: Maximum percentage gap - cost (Case A)

Maximum percentage gap — cost [%] Case A			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	99.97	99.97	99.97
Min value	0.01	0.00	0.04
Mean value	56.48	55.75	54.13
Median value	59.70	58.78	56.79
Mode	0.01	0.00	0.04
Standard deviation	28.17	26.93	27.35
IQR	47.97	44.10	45.31

We also compare the mean values of the maximum percentage gap statistically, for quality and cost criteria. The number of strictly positive maximum percentage gaps for the quality criteria over all instances, in (B.1) equals 1995, in (B.2) 1691 and in (B.3) 1619. For cost criteria, they equal 1064, 1670 and 1613 respectively.

Firstly we test the null hypothesis stated as: *The mean values of the strictly positive maximum percentage gap for the quality criteria are equal for any two of the problems considered*. We test this hypothesis against the alternative hypotheses stated below. In the points below we give also the calculated values of the test statistics.

1. The mean value of the strictly positive maximum percentage gap for the quality criteria for (B.1) is greater than for (B.2), $W = 55.42$;

2. The mean value of the strictly positive maximum percentage gap for the quality criteria for (B.1) is greater than for (B.3), $W = 76.18$;

3. The mean value of the strictly positive maximum percentage gap for the quality criteria for (B.2) is greater than for (B.3), $W = 9.49$;

Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means of the maximum percentage gap for the quality criteria, and accept all of the above alternative hypotheses.

Secondly, we perform the test for the cost criteria as well. We test the following null hypothesis: *The mean values of the strictly positive maximum percentage gap for the cost criteria are equal for any two of the problems considered*, against the alternative hypotheses given in points below. Similarly to the previous test, we also give the resulting test statistics.

1. The mean value of the strictly positive maximum percentage gap for the cost criteria for (B.1) is greater than for (B.2), $W = 0.68$;

2. The mean value of the strictly positive maximum percentage gap for the cost criteria for (B.1) is greater than for (B.3), $W = 2.14$;

3. The mean value of the strictly positive maximum percentage gap for the cost criteria for (B.2) is greater than for (B.3), $W = 1.71$.

Test statistics for test no. 2 and 3 fall within the rejection region of H_0 for $\alpha = 0.05$. Thus, we can accept the alternative hypotheses that the mean value of maximum percentage gaps for cost criterion obtained by solving the single-criterion problem was significantly higher than by solving the multi-criteria equitable problem. Moreover, the equitable problem also managed to produce results with lower mean value of the same than the non-equitable problem. Yet, no statistically significant difference was seen for test no. 1 and as such not enough evidence was collected to reject H_0 in this test.

4.3.4 Mean value of the utility function

As described in Sec. 3.4.5, achievement functions used in the Reference Point Method aggregation may be understood as the numerical value of the Decision Maker's satisfaction on reaching a given value of the criterion. In that sense, they can be understood as specific utility functions of the same. Therefore, we can directly use them to compare the obtained results as they represent the sentiment of the customers on the obtained dispatch results. Of course, the higher the value, the more is a customer satisfied with the criterion result they obtained.

The values of the functions are directly calculated using (3.7) with parameters as specified in Sec. 4.2. For comparisons, we take the mean value of the utility taken over both quality/cost criteria combined per customer. Then, for each test instance, the mean of the mean is reported as the index under consideration. The logic behind such a representation is that it shows the average sentiment on meeting the requirements of an average customer participating in the dispatching process. As we know, we require for SGIs that both requirements for cost and quality are met to the greatest extent for as many customers as possible. Thus, comparing the obtained results in that combined way gives information on the general sentiment of the complete system of customers.

The results of the comparison are shown in the box plot in Fig. 4.4, as well as in Tab. 4.6.

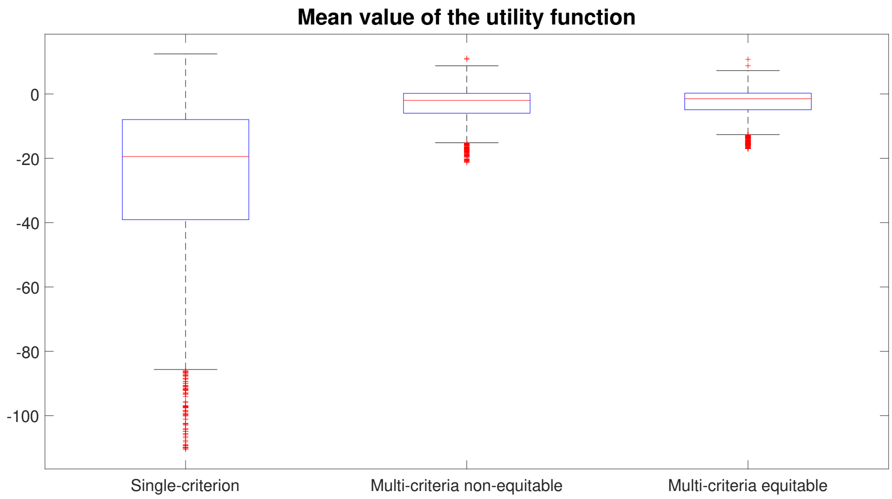


Figure 4.4: Mean value of the utility function (Case A)

Table 4.6: Mean value of the utility function (Case A)

Mean value of the utility function Case A			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	1.87×10^3	1.87×10^3	1.87×10^3
Min value	-6.44×10^4	-9.24×10^3	-9.24×10^3
Mean value	-160.57	-34.83	-32.58
Median value	-25.31	-3.37	-2.54
Mode	-6.44×10^4	-9.24×10^3	-9.24×10^3
Standard deviation	1.52×10^3	297.48	295.92
IQR	56.09	13.42	10.82

As can be noted in the box plot, the mean value of the utility function is much higher for the multi-criteria approaches than in the single-criterion. However, there is some high variability in the tabular data reported. There is some big difference between the mean and median values and large standard deviation values, which was not observed in previously studied indices. One may suspect that this is caused by highly influential data points representing some highly dissatisfied customers. Since the utility function is built in a way that the dissatisfaction has a much higher weight assigned than *oversatisfaction* ($\beta << \gamma$), such an influential behaviour can be explained and expected.

Thus, to grasp the trends in the experimental data, we present it with outliers also filled in Tab. 4.7.

Table 4.7: Mean value of the utility function (outliers filled, Case A)

Mean value of the utility function (outliers filled) Case A			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	12.45	11.12	10.77
Min value	-110.43	-21.27	-17.03
Mean value	-26.91	-3.53	-2.87
Median value	-19.44	-1.99	-1.50
Mode	-110.43	-21.27	-17.03
Standard deviation	24.78	4.99	4.23
IQR	31.12	6.17	5.17

As one can see, filling outliers made the tabular values represent the box plot. As we can notice, multi-criteria problems yield much more tailored to customers' needs results in terms of the mean value of the utility functions. Those observations are checked statistically below. This is done both for the data without outliers filled and with them filled.

We test the null hypothesis stated as: *The mean of the mean values of the utility function are equal for any two of the problems considered*. We test this hypothesis against the alternative hypotheses stated below. In the points below we give also the calculated values of the test statistics, where W is the statistic calculated on the data without filling outliers and W_o on the data with outliers filled.

1. The mean of the mean value of the utility function for (B.2) is greater than for (B.1), $W = 3.62, W_o = 41.37$;
2. The mean of the mean value of the utility function for (B.3) is greater than for (B.1), $W = 3.69, W_o = 42.77$;
3. The mean of the mean value of the utility function for (B.3) is greater than for (B.2), $W = 0.24, W_o = 4.53$.

The first two tests yield statistics which fall within the rejection of H_0 for $\alpha = 0.05$ for both the data with and without the outliers filled. Thus, at this α we can reject the null hypotheses on the equality of the means of the mean value of the utility function for the single-criterion and multi-criteria problems, regardless of the outliers. This is not the case for comparisons between the non-equitable, and equitable problems. In this case, we are only able to reject the null hypothesis when performing the test on the outlier-filled data.

To conclude, for Case A, the multi-criteria approach to dispatching of SGIs was able to yield dispatch such that an average customer is statistically significantly more satisfied with the obtained results. This is estimated by considering crafted utility functions.

4.3.5 Optimiser's solution time

The last index that we consider in our test is the solution time of the optimiser. The summary is presented in Fig. 4.5 and in Tab. 4.8.

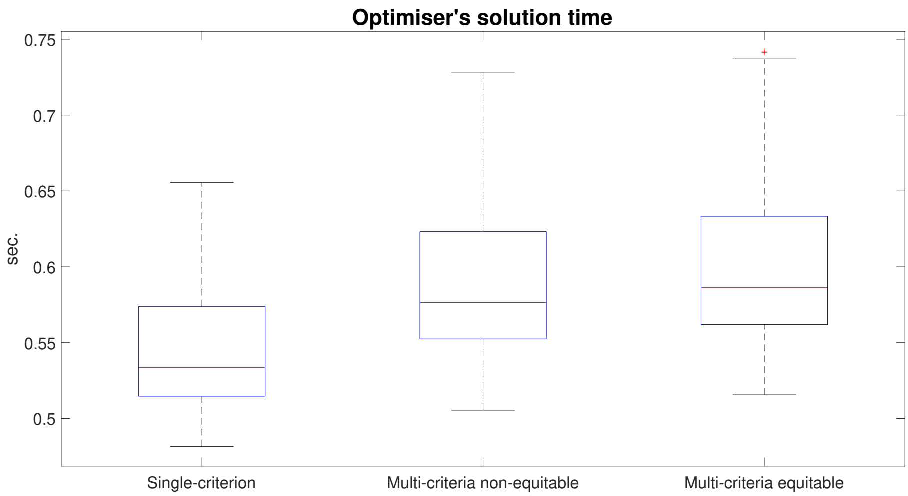


Figure 4.5: Optimiser's solution time (Case A).

Table 4.8: Optimiser's solution time (Case A)

Optimiser's solution time [sec.] Case A			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	1.34	1.35	1.40
Min value	0.48	0.51	0.52
Mean value	0.57	0.61	0.62
Median value	0.53	0.58	0.59
Mode	0.52	0.55	0.56
Standard deviation	0.10	0.10	0.10
IQR	0.08	0.08	0.08

One can suspect, by looking at the data, that, on average, the multi-criteria problems are slightly more computationally demanding than the single-criterion one, with the equitable problem being the most demanding. This is checked statistically with the mean values of the solution time compared. We test the null hypothesis stated as *The mean values of the optimiser's solution time are equal for any two of the problems considered*. We test this hypothesis against the alternative hypotheses stated below. In the points below, we also give the calculated values of the test statistics.

1. The mean value of the optimiser's solution time of (B.2) is greater than (B.1), $W = 12.76$;
2. The mean value of the optimiser's solution time of (B.3) is greater than (B.1), $W = 15.68$;
3. The mean value of the optimiser's solution time of (B.3) is greater than (B.2), $W = 2.94$.

All of the test statistics fall into the rejection region of H_0 at $\alpha = 0.05$. Therefore, we can reject the hypothesis of equality of mean values of the solution time of all problems and accept the alternative hypotheses stated above. Having said the above, we can conclude that the use of multi-criteria optimisation problems in the SGI dispatching introduces some additional computational burden. This can be expected, as the complexity of the models is higher due to additional constraints introduced for the computation of the achievement functions.

Similarly, due to introducing additional constraints, the mean solution time for the equitable problem is observed to be significantly higher than for the non-equitable one.

4.3.6 Conclusions and discussion — Case A

We have performed numerical experiments over 2,000 randomly generated instances of Case A with 10 customers and 13 suppliers/resources to be dispatched. Each customer had one cost criterion and one quality criterion linked to it. The tests were designed so to check if the use of multi-criteria approaches to the SGI dispatching adds functional benefits. We assumed in the tests that reservations and aspirations are estimated using expert knowledge/guidelines and that they reflect correctly the needs of the participants.

In the test, we have shown that, when multi-criteria approaches are applied, the criteria significantly more often meet (or even outperform) their reservations.

When looking deeper into the outcomes, the test has shown that the multi-criteria approaches considered, allowed for significantly reducing the number of quality outcomes not reaching their reservation by at least 10%. This, was however not visible for the cost criteria, where this number increased for the multi-criteria problems. Yet, this increase was not as important as the decrease in the quality ones.

Moreover, for quality criteria, the strictly positive maximum percentage gaps were significantly higher for the single-criterion approach. Yet, for cost criteria, the only significant difference was shown when the equitable problem was applied.

All the above observations were also confirmed through investigation of the mean value of the utility function, formulated as the RPM achievement function. We have shown, that in Case A, significantly higher mean values of the utility function were obtained by solving the multi-criteria problems, as opposed to the single-criterion one.

Comparisons between the non-equitable and the equitable problems showed that the equitable one can perform better. This is because it follows the principle of transfers, and as such reduces inequalities between participants. In that sense, the results are more *packed* and more often reach their reservations.

Overall, we may say that thanks to consideration of the quality criteria, in addition to the cost criteria can make the dispatching results more tailored to the customers' needs. Yet, this came at significantly increasing the optimiser's solution time.

4.4 Results — Case B

The setup of this test case is summarised in the points below. Similarly to the tests performed in Case A, each of the calculations is performed three times - by solving problems (B.1), (B.2) and (B.3), which obviously results in three different Pareto-optimal solutions for each instance.

Test setup:

- Number of customers $|\mathcal{P}_k| = 50$;
- Number of suppliers/resources $|\mathcal{P}_s| = 75$;
- Number of criteria for each $i \in \mathcal{P}_k$ = 4 (two cost criteria, two quality criteria), i.e., total number of criteria equals 200;
- Randomly generated aspirations, reservations, $d^j_{i,n}$ and $t^j_{i,k}$;
- In total 2,000 instances were generated, solved and compared.

This Section describes how these solutions impact the indices considered in Sec.4.2.1 as estimated through statistical analyses.

The methods applied to Case B are exactly the same as those applied previously to Case A.

4.4.1 Number of criteria being at least as good as reservations

This Section presents statistically the number of criteria being as good (or better than) their respective reservation, obtained for Case B. Descriptive statistics are shown in Tab. 4.9 and the box plot in Fig. 4.6.

Figure 4.6: Number of criteria at least as good as their reservations (Case B).

Table 4.9: Number of criteria being at least as good as reservations (Case B)

Number of criteria being at least as good as reservations Case B			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	161.00	190.00	193.00
Min value	121.00	135.00	120.00
Mean value	142.82	169.84	173.87
Median value	143.00	170.00	174.00
Mode	142.00	171.00	173.00
Standard deviation	5.63	7.19	6.25
IQR	5.00	10.00	8.00

By looking at the data, one can immediately recognise that the pattern from Case A is kept. This means that the number of criteria being at least as good as their reservation is much higher in the multi-criteria approaches than in the single criterion one. Similarly, the equitable problem seems to produce better results than the non-equitable one. These observations are verified statistically below.

We verify the null hypothesis stated as: *The mean values of the number of criteria being at least as good as the reservation levels are equal for any two problems considered* and the alternative hypotheses are stated in points below. These points also specify the resulting test statistics (4.2).

1. The mean value of the number of criteria being at least as good as the reservation level in (B.2) is greater than (B.1), $W = 132.36$;

2. The mean value of the number of criteria being at least as good as the reservation level in (B.3) is greater than (B.1), $W = 165.04$;

3. The mean value of the number of criteria being at least as good as the reservation level in (B.3) is greater than (B.2), $W = 18.90$.

The statistical tests confirm the observations, as all of the statistics fall into the rejection region of H_0 at $\alpha = 0.05$. We can thus say that the multi-criteria approaches are able to produce results, with a significantly higher number of criteria being at least as good as their reservation.

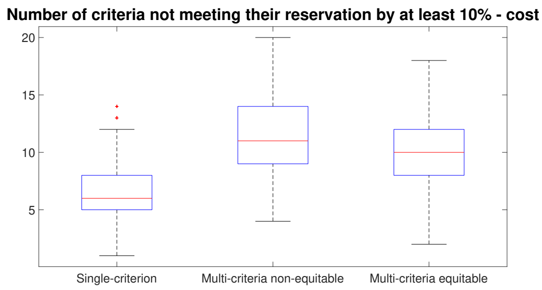
It is worth noting that with the increase in the number of suppliers and customers, these differences appear even higher. This can be seen by comparing them with the ones received for Case A.

4.4.2 Number of criteria not meeting their reservations by at least 10%

Similarly to Case A, we investigate the impact of the methods on the number of criteria which got assigned with a value worse by at least 10% than their reservation.

The summary of results obtained is shown in Tab. 4.10 and in Tab. 4.11, as well as in Fig. 4.7.

(a) quality



(b) cost

Figure 4.7: Number of criteria not meeting their reservations by at least 10% (Case B)

Table 4.10: Number of quality criteria not meeting their reservations by at least 10% (Case B)

Number of quality criteria not meeting their reservations by at least 10% Case B			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	64.00	33.00	31.00
Min value	31.00	1.00	0.00
Mean value	44.99	8.30	6.53
Median value	45.00	8.00	6.00
Mode	46.00	8.00	6.00
Standard deviation	4.90	3.54	2.69
IQR	6.00	4.00	3.00

Table 4.11: Number of cost criteria not meeting their reservations by at least 10% (Case B)

Number of cost criteria not meeting their reservations by at least 10% Case B			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	17.00	28.00	44.00
Min value	1.00	2.00	2.00
Mean value	6.52	11.74	10.28
Median value	6.00	12.00	10.00
Mode	6.00	11.00	10.00
Standard deviation	2.49	3.70	3.42
IQR	3.00	5.00	4.00

One can see that when it comes to meeting quality criteria, the multi-criteria approaches perform importantly better in terms of the index studied. However, this is not the case for the cost criteria. In this case, the single-criterion performs better than the other ones considered. However, one can clearly see in the data, that the difference for cost criteria is not as high as for the quality ones. Similarly, the equitable problem allowed to produce results with a lower number of criteria not meeting their reservation by at least 10% than the non-equitable one. This observation is in line with the results obtained for the previously studied Case A.

The above observations are tested statistically. Firstly, we give the test for the null hypothesis stated as: *The mean values of number of quality criteria not meeting their reservations by at least 10% are equal for any two of the problems considered*. The alternative hypotheses, with the resulting test statistics, are given in the points below.

1. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.1) is greater than in (B.2), $W = 271.39$;

2. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.1) is greater than in (B.3), $W = 307.64$;

3. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.2) is greater than in (B.3), $W = 17.89$.

Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means of the number of quality criteria being worse than their reservations by at least 10%, and accept all of the above alternative hypotheses.

Secondly, we give the comparisons for the null hypothesis stated as: *The mean values of number of cost criteria not meeting their reservations by at least 10% are equal for any two of the problems considered*. The alternative hypotheses, with the resulting test statistics, are given in the points below.

1. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.2) is greater than in (B.1), $W = 52.32$;

2. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.3) is greater than in (B.1), $W = 39.72$;

3. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.2) is greater than in (B.3), $W = 12.97$.

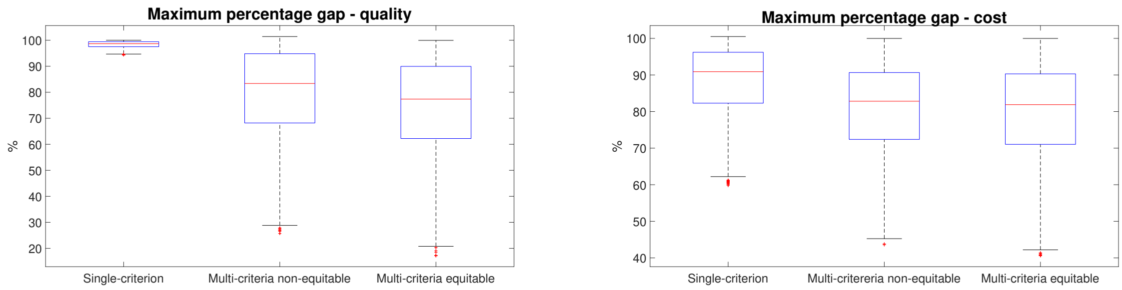
Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means of the number of cost criteria being worse than their reservations by at least 10%, and accept all of the above alternative hypotheses.

Similarly to what was observed for Case A, one may identify that the loss of performance for cost criteria is much lower than the gain in performance for quality criteria.

For this index, we can also observe, that with the increase of the number of suppliers/consumers, the differences in performance between the problems studied become more significant.

4.4.3 Maximum percentage gap

As in the previous case, we study the index of the maximum percentage gap between the reservation, and the obtained value of the criterion. In this Section we calculate the gap as previously — namely using (4.4), where only strictly positive gaps are considered. For each customer, we have two cost and two quality criteria. Thus, we report the maximum value for the cost criteria (over two criteria combined, over all customers), and for the quality accordingly the same. A summary of the results obtained is presented in Fig. 4.8 as well as in Tab. 4.12 and Tab. 4.13.



(a) quality (b) cost

Figure 4.8: Maximum percentage gap (Case B)

Table 4.12: Maximum percentage gap — quality (Case B)

Maximum percentage gap — quality [%] Case B			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	100.00	101.42	99.95
Min value	94.33	25.71	17.15
Mean value	98.32	79.48	73.96
Median value	98.66	83.38	77.37
Mode	94.33	25.72	17.15
Standard deviation	1.34	17.82	19.58
IQR	1.91	26.62	27.73

Table 4.13: Maximum percentage gap — cost (Case B)

Maximum percentage gap — cost [%] Case B			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	100.50	99.98	99.98
Min value	59.79	43.58	40.59
Mean value	88.31	80.53	79.34
Median value	90.89	82.83	81.88
Mode	59.79	43.58	40.59
Standard deviation	9.74	12.50	13.48
IQR	13.91	18.26	19.26

We also compare the mean values of the maximum percentage gap statistically, for quality and cost criteria separately. The number of strictly positive maximum percentage gaps for both the quality criteria over all instances and over all problems was 2,000.

Firstly, we test the null hypothesis stated as: *The mean values of the strictly positive maximum percentage gap for the quality criteria are equal for any two of the problems considered*. We test this hypothesis against the alternative hypotheses stated below. In the points below we also give the calculated values of the test statistics.

1. The mean value of the strictly positive maximum percentage gap for the quality criteria for (B.1) is greater than for (B.2), $W = 45.86$;

2. The mean value of the strictly positive maximum percentage gap for the quality criteria for (B.1) is greater than for (B.3), $W = 54.53$;

3. The mean value of the strictly positive maximum percentage gap for the quality criteria for (B.2) is greater than for (B.3), $W = 9.05$.

Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means of the maximum percentage gap for the quality criteria, and accept all of the above alternative hypotheses.

Secondly, we perform the test for the cost criteria as well. We test the following null hypothesis *The mean values of the strictly positive maximum percentage gap for the cost criteria are equal for any two of the problems considered*, against the alternative hypotheses given in points below. Similarly to the previous test, we also give the resulting test statistics.

1. The mean value of the strictly positive maximum percentage gap for the cost criteria for (B.1) is greater than for (B.2), $W = 14.93$;

2. The mean value of the strictly positive maximum percentage gap for the cost criteria for (B.1) is greater than for (B.3), $W = 16.85$;

3. The mean value of the strictly positive maximum percentage gap for the cost criteria for (B.2) is greater than for (B.3), $W = 2.30$.

All test statistics fall within the rejection region of H_0 for $\alpha = 0.05$. Thus, enough evidence was collected to state that the maximum gap for cost in Case B was significantly higher for the single-criterion approach than the multi-criteria ones. Moreover, significant difference also was revealed at the assumed significance between the means of the equitable and non-equitable multi-criteria problems for the cost criteria.

From the experiments performed, one can clearly state that multi-criteria problems allowed for producing SGI dispatching results, with a lower gap for the least favoured customer (maximum gap). Similarly to previous experiments, the differences were higher, compared to Case A. Thus, one may suspect that they do increase with a highe number of criteria, suppliers and customers considered.

4.4.4 Mean value of the utility function

In this Section, we report the mean values of the utility functions, for an average customer. We take the mean of each customer's values of the utility for each of their criteria and then take the mean of those. As a result, we obtain the average satisfaction of an average customer. The results are presented in the box plot in Fig.4.9 and in Tab.4.14

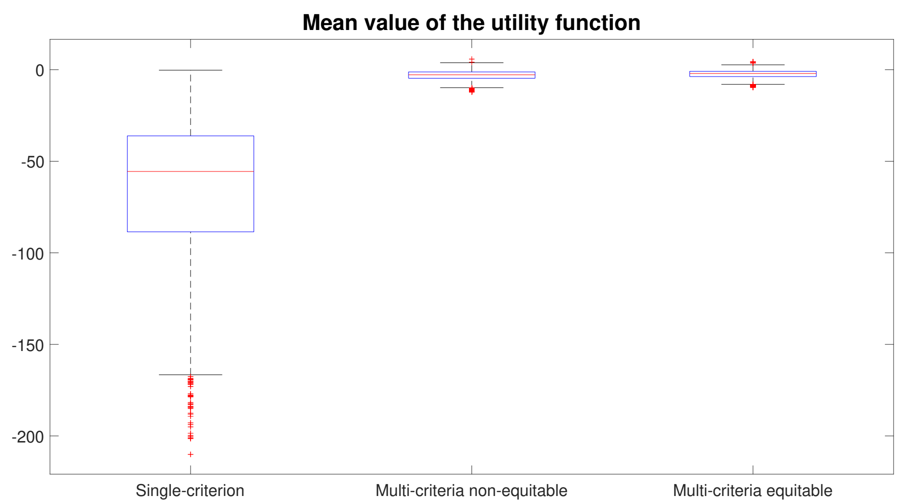


Figure 4.9: Mean value of the utility function (Case B)

Table 4.14: Mean value of the utility function (Case B)

Mean value of the utility function Case B			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	-10.40	147.74	144.20
Min value	-3.83×10^4	-7.84×10^3	-7.84×10^3
Mean value	-255.00	-15.73	-13.93
Median value	-62.62	-3.10	-2.36
Mode	-3.83×10^4	-7.84×10^3	-7.84×10^3
Standard deviation	1.66×10^3	249.62	249.18
IQR	84.62	4.70	3.82

Similarly to the situation observed for Case A, we can see some significant variability in the tabular data reported. Therefore, we also present the descriptive statistics calculated on the data with outliers filled in Tab. 4.15.

Table 4.15: Mean value of the utility function (outliers filled, Case B)

Mean value of the utility function (outliers filled) Case B			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	-0.24	5.80	4.42
Min value	-210.07	-12.18	-9.67
Mean value	-67.86	-3.27	-2.49
Median value	-55.57	-2.79	-2.06
Mode	-210.07	-12.18	-9.67
Standard deviation	42.26	2.78	2.25
IQR	52.41	3.45	2.86

Similarly to the previously studied case, filling outliers allowed for reducing the variability in the data. Again, this is much likely due to the removal of highly influential data points.

Statistically, we test the null hypothesis stated as: *The means of the mean values of the utility function are equal for any two of the problems considered*. We test this hypothesis against the alternative hypotheses stated below. In the points below we also give the calculated values of the test statistics, where W is the statistic calculated on the data without filling outliers and W_o on the data with outliers filled.

1. The mean of the mean value of the utility function for (B.2) is greater than for (B.1),

$W = 6.39, W_o = 68.20$;

2. The mean of the mean value of the utility function for (B.3) is greater than for (B.1), $W = 6.44, W_o = 69.08$;
3. The mean of the mean value of the utility function for (B.3) is greater than for (B.2), $W = 0.23, W_o = 9.76$.

The result of statistical inference was obtained exactly as for Case A. We observe a statistically significant difference in the mean value of the utility function between the results obtained by solving the single-criterion and multi-criteria problems. This is observed regardless of whether the outliers are filled or not. Similarly to Case A, a significant difference between results coming from the equitable and the non-equitable approach was seen only when the outliers were filled.

As seen for previously studied indices, the observed differences between results coming from single and multi-criteria problems were greater than in Case A. This may lead to thinking that with the increase in case complexity, the benefit of dispatching the SGIs with multi-criteria problems is higher.

4.4.5 Optimiser's solution time

The last index that we consider in our test is the solution time of the optimiser. The summary is presented in Fig. 4.10 and in Tab. 4.16.

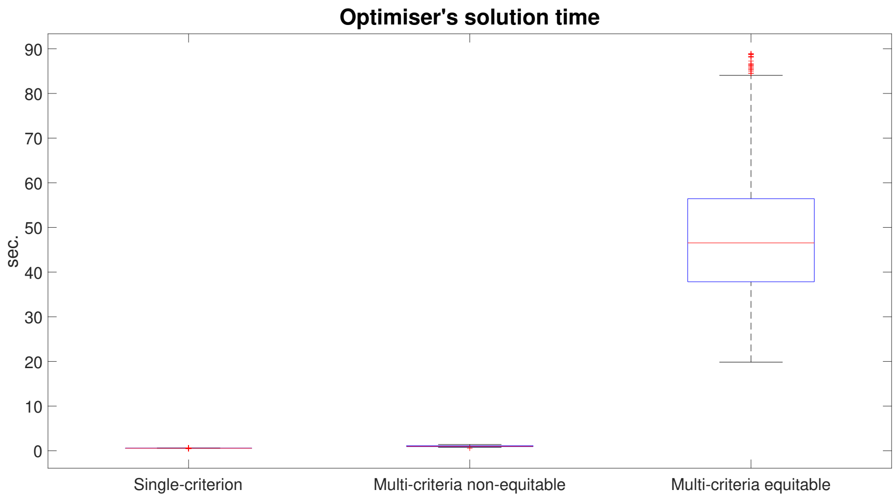


Figure 4.10: Optimiser's solution time (Case B).

Table 4.16: Optimiser's solution time (Case B)

Optimiser's solution time [sec.] Case B			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	1.59	3.16	2.87×10^4
Min value	0.52	0.79	19.84
Mean value	0.60	1.08	80.47
Median value	0.58	1.00	46.80
Mode	0.56	0.79	19.84
Standard deviation	0.09	0.23	908.83
IQR	0.03	0.23	19.34

As we can see in the box plot, which is drawn just like all other box plots reported — on the data with outliers filled, on average the solution time of the multi-criteria equitable problem is visibly higher than for the other problems considered. Despite, the fact that it is visibly higher, one can notice that the tabular values reported highly variable for the equitable problem. Thus, it may be due to some outliers in time measurement data or due to some particular difficult test instances. This is why to understand better the trends in the data we fill the outliers for the three groups considered and report such data in Tab. 4.17.

Table 4.17: Optimiser's solution time (uutliers filled, Case B)

Optimiser's solution time [sec.] (outliers filled) Case B			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	0.63	1.40	88.93
Min value	0.52	0.57	19.84
Mean value	0.57	1.04	47.98
Median value	0.57	0.99	46.54
Mode	0.56	0.57	19.84
Standard deviation	0.02	0.14	13.57
IQR	0.02	0.19	18.60

The observation of the data suggests that application of multi-criteria problems was more computationally demanding than the single-criterion one, with the equitable problem being extremely demanding. This observation is checked statistically. Similarly to studying the mean value of the utility function, we perform the tests both on the data with and without the outliers filled. We test the null hypothesis stated as *The mean values of the optimiser's solution time are equal for any two of the problems considered*. We test this hypothesis against the alternative hypotheses stated below. In the points below we also give the calculated values of the test statistics, where W — statistic calculated without filling outliers, W_o — statistic calculated on data with outliers filled.

1. The mean value of the optimiser's solution time of (B.2) is greater than in (B.1), $W = 87.57, W_o = 151.20$;

2. The mean value of the optimiser's solution time of (B.3) is greater than in (B.1), $W = 3.93, W_o = 156.25$;

3. The mean value of the optimiser's solution time of (B.3) is greater than in (B.2), $W = 3.91, W_o = 154.73$.

All of the test statistics fall into the rejection region of H_0 at $\alpha = 0.05$, regardless of whether they are calculated on data with outliers filled or not. Therefore, we can reject the hypothesis of equality of mean values of the solution time of all problems, and accept the alternative hypotheses stated above. Having said the above, we can conclude that the use of multi-criteria optimisation problems in the SGI dispatching introduced additional computational burden in Case B, just as it did for Case A.

Yet, one should notice that the mean solution time of the equitable problem was more than one order of magnitude higher than that of the other problems. Such a huge increase might possibly cause problems in the SGI dispatching, where the dispatcher has to make quick and accurate decisions.

4.4.6 Conclusions and discussions — Case B

In this Section, we performed numerical experiments on Case B, which was characterised by a larger number of customers (50), suppliers (75) and also criteria per each customer (2 for cost and 2 for quality), than in the previously studied Case A. We randomly generated 2,000 test instances, which were then analysed statistically in order to answer the question, of whether adding new quality-based criteria adds value to the dispatching, allowing to yield more tailored results to the customers' needs. Similarly to the previously studied Case, we have assumed that the aspirations/reservations correctly reflect customers' needs and in that sense can be used as a basis for the analyses.

In experiments performed for Case B, we have confirmed all observations from Case A, namely that on average significantly more criteria are better than their reservations when the multi-criteria approach is applied. Similarly, on average a significantly lower number of criteria are worse for quality by more than 10%, which is not necessarily the case for cost criteria when the quality-based criteria are considered. Yet, the gain in performance for the quality ones is much higher, than its loss for the cost. This is even more compensated when we look at the mean value of utility, where we have shown that it is significantly higher when the multi-criteria approaches are applied. Similarly, the maximum percentage gaps are lower when the multi-criteria problems are applied, regardless if this is for quality or for cost.

Our test revealed, that considering quality-based criteria in the SGI dispatch process added value to the SGI dispatch problems in Case B, as opposed to the classical cost-minimal approach. Yet, this came at the cost of increasing the computational burden of the problems, with the equitable problem being solved in tremendously long average times. Times as in the multi-criteria equitable problem can be too long for the SGI dispatch optimisation problems, where the dispatcher needs to make quick and accurate decisions.

All indices are further investigated in experiments applied to Case C.

4.5 Results — Case C

The setup of this test case is summarised in the points below. Similarly, to tests performed in both previous cases, each of the calculations is performed three times — by solving problems (B.1), (B.2) and (B.3), which results in three different Pareto-optimal solutions for each instance.

Test setup:

- Number of customers $|\mathcal{P}_k| = 100$;
- Number of suppliers/resources $|\mathcal{P}_s| = 120$;
- Number of criteria for each $i \in \mathcal{P}_k$ = 4 (two cost criteria, two quality criteria), i.e., total number of criteria equals 400;
- Randomly generated aspirations, reservations, $d^j_{i,n}$ and $t^j_{i,k}$;
- In total 2,000 instances were generated, solved and compared.

This Section describes how these solutions impact the indices considered in Sec. 4.2.1 as estimated through statistical analyses.

The methods applied to Case C are exactly the same as those applied previously to Cases A and B.

4.5.1 Number of criteria being at least as good as reservations

Wen apply the same approach as in previously discussed Cases. In that sense, we first show the box plot in Fig. 4.11, show the data in tabular form in Tab. 4.18 and then compare it statistically further.

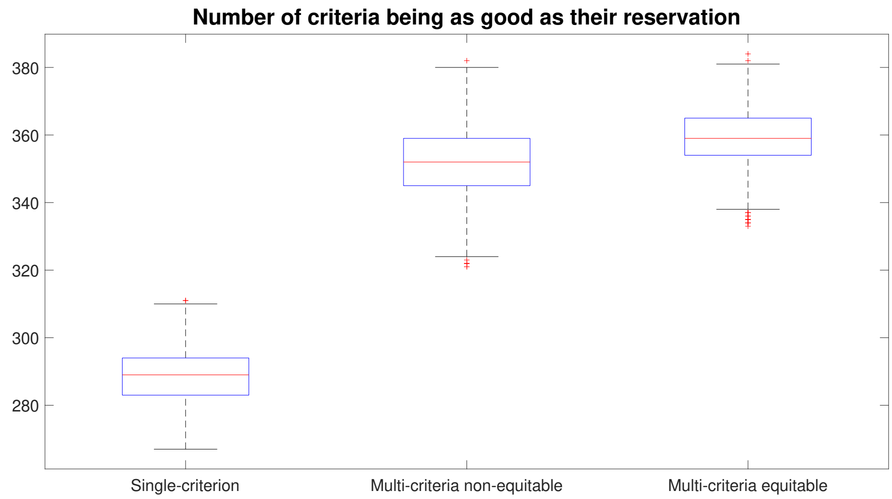


Figure 4.11: Number of criteria being at least as good as their reservations (Case C).

Table 4.18: Number of criteria being at least as good as reservations (Case C)

Number of criteria being at least as good as reservations Case C			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	314.00	382.00	384.00
Min value	263.00	209.00	0.00
Mean value	288.73	350.94	358.72
Median value	289.00	352.00	359.00
Mode	287.00	354.00	359.00
Standard deviation	7.97	11.93	11.51
IQR	11.00	14.00	11.00

One may see in the visual presentation of the data that the patterns seen in previous Cases are conserved. This means that multi-criteria problems allowed for generation of results with higher number of criteria being at least as good as their reservations, with the equitable problem performing slightly better. Yet, the minimum value of index for the equitable problem is really low — equal to zero. However, the overall trend is as seen before.

We check the observation statistically. For this, we verify the null hypothesis stated as: *The mean values of the number of criteria being at least as good as the reservation levels are equal for any two problems considered* and the alternative hypotheses are stated in points below. These points also specify the resulting test statistics (4.2).

1. The mean value of the number of criteria being at least as good as the reservation in (B.2) is greater than (B.1), $W = 193.93$;

2. The mean value of the number of criteria being at least as good as the reservation in (B.3) is greater than (B.1), $W = 223.66$;

3. The mean value of the number of criteria being at least as good as the reservation in (B.3) is greater than (B.2), $W = 20.99$.

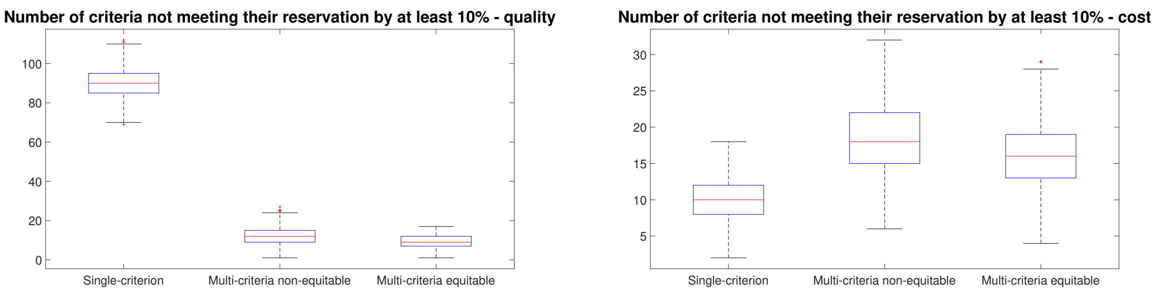
The statistical tests confirm the observations, as all of the statistics fall in the rejection region of H_0 at $\alpha = 0.05$. We can thus say that the multi-criteria approaches are able to produce results, with a significantly higher number of criteria being at least as good as their reservation.

Similarly to what has been observed previously, with the increase of case complexity the tests statistics also increased (as compared with Case B and Case A). This pattern was also observed while comparing results from Case B with Case A.

4.5.2 Number of criteria not meeting their reservations by at least 10%

Similarly to previous Cases, we investigate the impact of the methods on the number of criteria which got assigned with a value worse by at least 10% than their reservation.

The summary of results obtained is shown in Tab. 4.19 and in Tab. 4.20, as well as in Fig. 4.12.



(a) quality (b) cost

Figure 4.12: Number of criteria not meeting their reservations by at least 10%(Case C)

Table 4.19: Number of quality criteria not meeting their reservations by at least 10% (Case C)

Number of quality criteria not meeting their reservations by at least 10% Case C			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	114.00	84.00	22.00
Min value	69.00	1.00	0.00
Mean value	89.92	13.19	9.63
Median value	90.00	12.00	9.00
Mode	89.00	11.00	10.00
Standard deviation	7.02	6.12	3.23
IQR	10.00	6.00	5.00

Table 4.20: Number of cost criteria not meeting their reservations by at least 10% (Case C)

Number of cost criteria not meeting their reservations by at least 10% Case C			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	25.00	92.00	33.00
Min value	1.00	4.00	0.00
Mean value	10.29	19.02	15.99
Median value	10.00	19.00	16.00
Mode	10.00	19.00	17.00
Standard deviation	3.22	5.74	4.38
IQR	4.00	7.00	6.00

Similarly to the previously studied Cases, the multi-criteria approaches allowed to produce visibly better results in terms of the quality index. This is however, not seen in the cost index, where the single-criterion approach performs better. Yet, the difference for cost is not as visible as the difference for quality. Similarly to the previous Cases, the equitable approach performs better for both indices than the non-equitable one.

Now, we test the above observations statistically. Firstly, we give the test for the null hypothesis stated as: *The mean values of number of quality criteria not meeting their reservations by at least 10% are equal for any two of the problems considered*. The alternative hypotheses, with the resulting test statistics, are given in the points below.

1. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.1) is greater than in (B.2), $W = 368.30$;

2. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.1) is greater than in (B.3), $W = 464.42$;

3. The mean value of the number of quality criteria not meeting their reservations by at least 10% in (B.2) is greater than in (B.3), $W = 22.99$.

Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means of the number of quality criteria being worse than their reservations by at least 10%, and accept all of the above alternative hypotheses.

Secondly, we give the comparisons for the null hypothesis stated as: *The mean values of number of cost criteria not meeting their reservations by at least 10% are equal for any two of the problems considered*. The alternative hypotheses, with the resulting test statistics, are given in the points below.

1. The mean value of the number of cost criteria not meeting their reservations by at least 10% in (B.2) is greater than in (B.1), $W = 59.32$;

2. The mean value of the number of cost criteria not meeting their reservations by at least 10% in (B.3) is greater than in (B.1), $W = 46.87$;

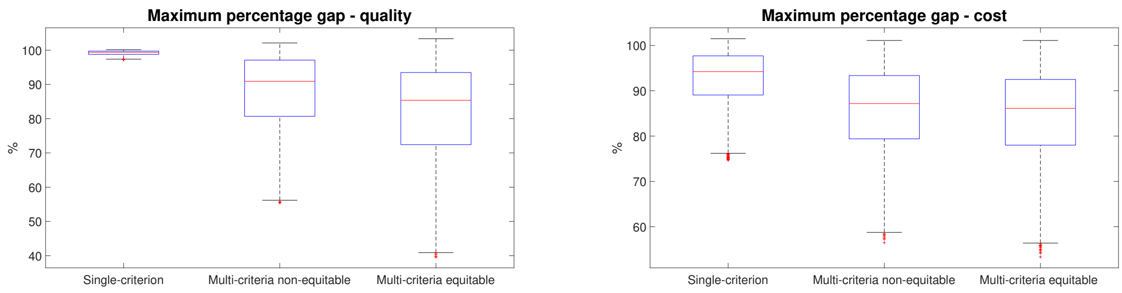
3. The mean value of the number of cost criteria not meeting their reservations by at least 10% in (B.2) is greater than in (B.3), $W = 18.78$.

Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means of the number of cost criteria being worse than their reservations by at least 10%, and accept all of the above alternative hypotheses.

Similarly to what was observed previously, one may identify that the loss of performance for cost criteria is much lower than the gain in performance for quality criteria. Moreover, we can also see that with the increase of the number of suppliers/consumers, the differences in performance between the problems studied become more significant — just as noticed in previous experiments.

4.5.3 Maximum percentage gap

In this Section we give the results for the maximum percentage gap calculated using (4.4), where only strictly positive gaps are considered. Similarly to Case B, for each customer, we have two cost and two quality criteria. We give the maximum value for the cost criteria (over two criteria combined, over all customers), and for the quality accordingly the same. A summary of the results obtained is presented in Fig. 4.13 as well as in Tab. 4.21 and Tab. 4.22.



(a) quality (b) cost

Figure 4.13: Maximum percentage gap (Case C)

Table 4.21: Maximum percentage gap — quality (Case C)

Maximum percentage gap — quality [%] Case C			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	100.00	100.00	99.98
Min value	93.03	21.75	18.22
Mean value	99.01	85.69	80.14
Median value	99.31	90.25	84.70
Mode	10.00	21.75	18.22
Standard deviation	93.04	14.33	16.74
IQR	1.08	18.33	22.76

Table 4.22: Maximum percentage gap — cost (Case C)

Maximum percentage gap — cost [%] Case C			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	100.00	100.00	99.94
Min value	19.87	36.25	36.25
Mean value	91.45	85.10	83.96
Median value	93.80	87.05	85.93
Mode	19.87	36.25	36.25
Standard deviation	8.45	10.45	10.89
IQR	9.40	14.34	14.90

From visual comparison of the data, one may see that in both indices the multi-criteria approaches performed better in terms of the mean value of the maximum percentage gap. This is in line with findings from Case B, yet no significant differences were revealed for cost criteria in Case A.

The above observations are compared statistically. The number of strictly positive gaps for the single-criterion and the non-equitable problems for both cost and quality are equal to 2,000, whereas for the equitable one to 1,999 for both cost and quality.

Firstly, we test the null hypothesis stated as: *The mean values of the strictly positive maximum percentage gap for the quality criteria are equal for any two of the problems considered*. We test this hypothesis against the alternative hypotheses stated below. In the points below we also give the calculated values of the test statistics.

1. The mean value of the strictly positive maximum percentage gap for the quality criteria for (B.1) is greater than for (B.2), $W = 41.45$;

2. The mean value of the strictly positive maximum percentage gap for the quality criteria for (B.1) is greater than for (B.3), $W = 50.31$;

3. The mean value of the strictly positive maximum percentage gap for the quality criteria for (B.2) is greater than for (B.3), $W = 11.27$.

Test statistics in all of the above tests fall within the rejection region of H_0, for significance level $\alpha = 0.05$. Therefore, at this α we can reject the null hypotheses on the equality of means of the maximum percentage gap for the quality criteria, and accept all of the above alternative hypotheses.

Secondly, we perform the test for the cost criteria as well. We test the following null hypothesis: *The mean values of the strictly positive maximum percentage gap for the cost criteria are equal for any two of the problems considered*, against the alternative hypotheses given in points below. Similarly to the previous test, we also give the resulting test statistics.

1. The mean value of the strictly positive maximum percentage gap for the cost criteria for (B.1) is greater than for (B.2), $W = 21.13$;

2. The mean value of the strictly positive maximum percentage gap for the cost criteria for (B.1) is greater than for (B.3), $W = 24.28$;

3. The mean value of the strictly positive maximum percentage gap for the cost criteria for (B.2) is greater than for (B.3), $W = 3.36$.

All test statistics fall within the rejection region of H_0 for $\alpha = 0.05$. Thus, enough evidence was collected to reject the null hypothesis on the equality of means of maximum percentage gap for cost criteria and to accept the alternative hypotheses stated above.

We can see that as in previous Case B, the multi-criteria approaches allowed to produce results with better mean values of the indices studied, with the equitable problem performing better than the non-equitable. The differences in means for cost criteria were higher than in the Cases A and B, yet this phenomenon was not observed for the quality criteria gaps.

4.5.4 Mean value of the utility function

In this Section, we report the mean values of the utility functions, for an average customer. We calculate them exactly as in Case B, namely take the mean of each customer's values of the utility for each of their criteria and then take the mean of those. As a result, we obtain the average satisfaction of an average customer. The results are presented in the box plot in Fig.4.14 and in Tab.4.23

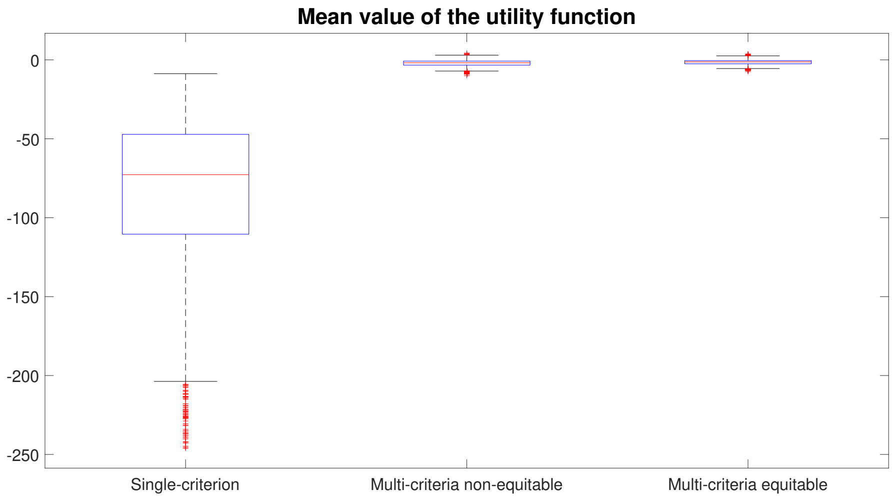


Figure 4.14: Mean value of the utility function (Case C)

Table 4.23: Mean value of the utility function (Case C)

Mean value of the utility function Case C			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	116.99	289.90	270.59
Min value	-3.83×10^5	-2.69×10^3	-2.68×10^3
Mean value	-461.41	-9.96	-7.29
Median value	-81.09	-2.19	-1.49
Mode	-3.83×10^5	-2.69×10^3	-2.68×10^3
Standard deviation	8.73×10^3	86.94	81.32
IQR	97.42	3.66	2.82

Again, high variability in the data is observed. We believe that this is due to some high influential observations. For this reason, we fill the outliers and report the data. This is given in Tab. 4.24.

Table 4.24: Mean value of the utility function (outliers filled, Case C)

Mean value of the utility function (outliers filled) Case C			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	-8.73	4.30	3.78
Min value	-246.13	-10.06	-7.47
Mean value	-85.66	-2.29	-1.60
Median value	-72.72	-1.87	-1.28
Mode	-246.13	-10.06	-7.47
Standard deviation	49.79	2.12	1.67
IQR	63.23	2.53	2.05

Similarly to the previous Cases, one may see that the mean value of the utility function is significantly higher for the multi-criteria approaches. This is tested statistically below both for data with outliers filled and without them filled.

We test the null hypothesis stated as: *The means of the mean values of the utility function are equal for any two of the problems considered*. We test this hypothesis against the alternative hypotheses stated below, where W is the statistic calculated on the data without filling outliers and W_o on the data with outliers filled.

1. The mean of the mean value of the utility function for (B.2) is greater than for (B.1), $W = 2.31, W_o = 74.81$;
2. The mean of the mean value of the utility function for (B.3) is greater than for (B.1), $W = 2.33, W_o = 75.45$;
3. The mean of the mean value of the utility function for (B.3) is greater than for (B.2), $W = 1.00, W_o = 11.31$.

All tests performed on data with outliers filled revealed the statistically significant difference of the mean values of the utility function (at $\alpha = 0.05$). Thus, for those tests we can reject the null hypotheses, accepting the alternative ones. However, no significant difference was revealed between the mean value of the utility function obtained by solving the equitable and the non-equitable problems on the data with outliers.

As seen for previously studied indices, the observed differences between results coming from single and multi-criteria problems were greater than in the two previous Cases. This further confirms the thinking that with the increase in case complexity, the benefit of dispatching the SGIs with multi-criteria problems is higher as opposed to doing so with the single-criterion one.

4.5.5 Optimiser's solution time

We conclude our investigations of Case C by looking at the solution time obtained by solving the three problems considered. The summary of results is presented in Fig. 4.15 and in Tab. 4.25.

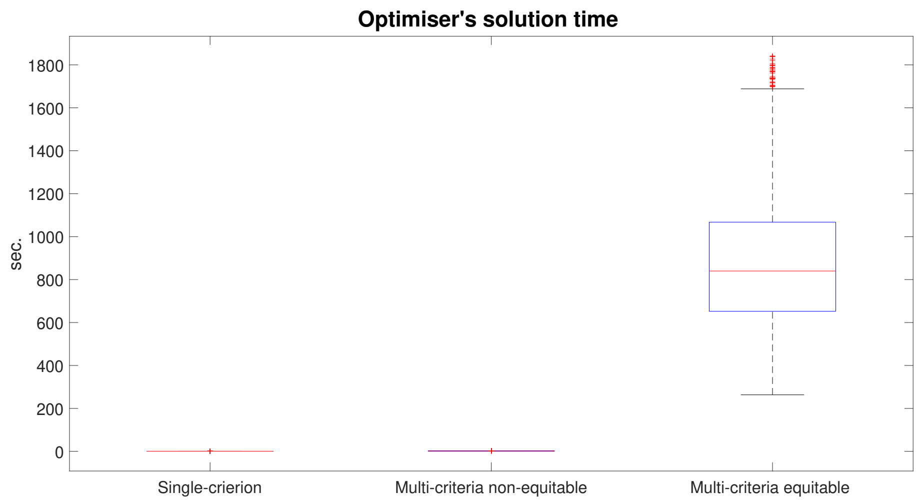


Figure 4.15: Optimiser's solution time (Case C).

Table 4.25: Optimiser's solution time (Case C)

Optimiser's solution time [sec.] Case C			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	1.32	6.43	4.55×10^4
Min value	0.54	1.23	263.31
Mean value	0.59	1.95	1.01×10^3
Median value	0.58	1.76	863.36
Mode	0.56	2.13	263.31
Standard deviation	0.05	0.56	1.27×10^4
IQR	0.04	0.47	454.74

One may also observe some high variability in the measurement data for the non-equitable problem. This may be due to some particularly difficult test instances. To compensate for this effect, to allow for better outlining the trends in the data, we fill outliers and report such data in Tab. 4.26.

Table 4.26: Optimiser's solution time (outliers filled, Case C)

Optimiser's solution time [sec.] (outliers filled) Case C			
	Single-criterion	Multi-criteria non-eq.	Multi-criteria eq.
Max value	0.87	2.69	1.84×10^3
Min value	0.54	1.23	263.31
Mean value	0.58	1.81	881.22
Median value	0.58	1.73	839.93
Mode	0.56	2.13	263.31
Standard deviation	0.02	0.30	305.80
IQR	0.03	0.38	415.07

For both the data with outliers filled and without we can see that the equitable problem was being solved in much higher times than the other ones. Similarly, we can see that the solution time for the non-equitable problem was also higher than the single-criterion one, yet to a much lower extent. We test those observations statistically below.

We test the null hypothesis stated as *The mean values of the optimiser's solution time are equal for any two of the problems considered*. We test this hypothesis against the alternative hypotheses stated below. In the points below we also give the calculated values of the test statistics, where W — statistic calculated without filling outliers, W_o — statistic calculated on data with outliers filled.

1. The mean value of the optimiser's solution time of (B.2) is greater than in (B.1), $W = 107.12$, $W_o = 181.06$;

2. The mean value of the optimiser's solution time of (B.3) is greater than in (B.1), $W = 35.56$, $W_o = 128.79$;

3. The mean value of the optimiser's solution time of (B.3) is greater than in (B.2), $W = 35.52$, $W_o = 128.61$.

All of the test statistics fall into the rejection region of H_0 at $\alpha = 0.05$, regardless of whether they are calculated on data with outliers filled or not. Therefore, we can reject the hypothesis of equality of mean values of the solution time of all problems, and accept the alternative hypotheses stated above. Having said the above, we can conclude that the use of multi-criteria optimisation problems in the SGI dispatching introduced a significant additional computational burden in Case C just as in the two previously studied ones. As such, one may say that the size of the problem solved is of utmost importance, when it comes to the solution time.

However, just as in the previous experiments, the equitable problem's solution time was way higher than for the other ones considered. It is then doubtful if such computational time may be acceptable for the application to SGIs.

4.5.6 Conclusions and discussion — Case C

In this Section, we have studied the problem the behaviour of the generic SGI dispatch optimisation problem as applied to Case C, with 100 customers and 120 resources to be dispatched. In this Case, each customer had two cost and two quality criteria linked to it, giving in total 400 dispatching criteria. There were 2,000 test instances of the Case solved by using the three formulations — single-criterion (B.1), multi-criteria non-equitable (B.2) and the multi-criteria equitable (B.3). The dispatching results were shown in box plots and compared statistically.

The tests confirmed the observation seen in previous Cases, that in general by applying the multi-criteria formulations it is possible to much more often meet or outperform the reservations for the criteria. Moreover, a significantly lower number of the quality criteria were worse than their reservations by at least 10% when the multi-criteria problems are solved. Unfortunately, this comes at increasing this number for cost criteria. Yet, the mean value of the increase for cost is much less significant than its decrease for quality. Then, the multi-criteria formulations allow also for reducing the maximum percentage gap (for both quality and cost criteria) and for achieving a significantly higher mean value of the utility function taken over the customers. Our tests also revealed that the multi-criteria problems significantly reduced the mean value of the maximum percentage gap, regardless of whether the equitable or non-equitable formulations were applied. Amongst the multi-criteria approaches the equitable one allowed to produce better results for the above indices than the non-equitable one.

As it comes to the solution time, it was significantly higher for the multi-criteria ones than for the single-criterion formulation, with the equitable one tremendously more time from the others to be solved. However, the increase in time for the non-equitable one was still well-bearable for the SGI dispatching applications.

4.6 Summary

In this Chapter, we have performed numerical experiments on the generic SGI dispatch optimisation problem over three test Cases, each one with 2,000 randomly generated test instances. In that sense, in total 6,000 numerical experiments have been done. Their purpose was to answer the main research question on whether adding quality-based criteria to the generic Services of General Interest dispatch optimisation problem adds value to the dispatching, allowing to yield more tailored results to the customers' needs, on a synthetic (computer-generated) data set. In the experiments, we compared statistically the dispatching of SGIs using two multi-criteria problems (non-equitable and equitable) with the standard cost minimisation single-criterion optimisation problem. The multi-criteria problems considered were solved using the Reference Point Method scalarisation. In the test, we assumed that the reservations and aspirations reflect correctly customers' needs, as they either come from them directly or are derived from standards and/or guidelines (e.g., medical guidelines).

We have shown that applying the multi-criteria optimisation problems to the dispatching of SGIs allows to yield results, which give on average: higher mean values of customers' satisfaction, a higher number of criteria meeting or outperforming their reservation, lower number of quality criteria not meeting their reservation by at least 10% and lower maximum percentage gap for quality criteria. What is more for larger test Cases, the test has shown that the maximum percentage gap was on average smaller also for the cost criteria. For all tests we assumed the significance level equal to $\alpha = 0.05$. It is also worth noting that with the increase in case sizes, the improvements achieved by using the multi-criteria problems were even more visible.

An increase in performance of the above indices, when the multi-criteria problems were used, came also with decreasing the performance in the number of cost criteria not meeting their reservation (increasing their number). This decrease was however much less important than its increase in quality criteria. What is more, this was even further improved by solving the equitable problem. Unfortunately, the application of multi-criteria problems significantly increased the optimiser's solution time for larger problems. For both the equitable and non-equitable formulations the solution time was tractable — we managed to solve both of them to Pareto-optimality on the test laptop. Yet, the solution time of the equitable one was way too high to use it efficiently for the SGI dispatching in real life. On the contrary, the increase in computational burden was still bearable on the experimental computer for the non-equitable problem. As such we may say, that we experienced a trade-off between obtaining results, that generally were significantly more tailored to customers' needs (when the SAOQ problem was solved) and the optimiser's solution time.

One may however argue, that considering recent developments in decomposition methods of optimisation problems, combined with the increasing popularity of cloud/quantum computing resources, computational burden is no longer a real problem. Since studying this phenomenon does not help in answering the main research question, it is deemed out of scope for this very work.

From our tests, one may conclude, that a possible loss of performance of results, witnessed for the *functional* indices between the non-equitable and the equitable problems is not as important, as the difference in solution times between them. Thus, in the next Chapters we solve the non-equitable problem and not the equitable one — because of the high computational time. In real-life applications, however, it is a matter of making a deeply investigated trade-off between the solution time and the results obtained. This is to be carefully decided by the decision maker.

This study carried some limitations, mostly due to its theoretical and synthetic nature. Yet, we argue that thanks to that the results may be well generalised for multiple applications. In the next Chapters we investigate two of such possible applications — dispatching of electrical energy generating units and dispatching of EMS ambulances to patients, and patients to Emergency Departments in hospitals.

All in all, however the results of the experiments over three Cases with 2,000 instances each, allow us to confirm that adding quality-based criteria to the generic Services of General Interest dispatch optimisation problem (SDOQ) adds value to the dispatching, allowing to yield more tailored results to the customers' needs on the synthetic Cases considered.

Chapter 5

Case Study I: Electrical energy generation

In this Chapter, we put the SDOQ problem to action in the energy generation sector. We apply it in designing a decision support system (DSS) for dispatching of the electrical energy generating units on the market with a peer-to-peer offering system (with its Operator, acting as the dispatcher). We focus on a situation, where there may exist conscious customers, who are eager to pay more to receive energy from more ecological or socially responsible sources. In other words, we differentiate the product through certain quality criteria. This Chapter is completely based on our paper [13]. Some numerical experiments presented in this Chapter are extended or amended as opposed to the paper, yet many parts are cited directly as already published. In the first Sections of the Chapter, we set grounds for the particular problems of the peer-to-peer energy trade, with drawing the picture of why considering quality-based criteria may be interesting in this particular case. We also review the respective literature. Then, we outline the proposed balancing architecture. Next, the respective mathematical optimisation models are presented, which are followed by the analysis of a numerical Case Study. We finish the Chapter by giving conclusions, discussion and possible future research possibilities.

Please note, that indexing of variables and parameters might differ from the one given in previous Chapters. This is because the Case Study consists of more complex mathematical models with more variables and constraints (describing the set $\mathcal{Q}$ of the SDOQ problem), where indices must be re-introduced and re-defined. The indexing and variables presented in this Chapter are only applicable to this very one.

5.1 Introduction

Nowadays, there is a trend of shifting from centrally-controlled power systems, where control and trade actions are created by the system operators, towards more decentralised, consumer-oriented systems, where market participants may reveal more preferences and may have a greater impact on market operations. Prosumers are willing to participate in this transformation actively.

Due to the high penetration of distributed energy resources (DERs), such as prosumers and renewable energy sources (RES), the tasks of energy supply, dispatch of generating units and system balancing in the power grids are becoming more and more complex. High penetration of uncertain weather-related sources makes secure delivery even more difficult.

Therefore, appropriate market mechanisms allowing for direct participation of many actors and for mitigating risks should be developed. A tempting approach is the peer-to-peer (p2p) market [151], where peers trade energy directly with each other. Some real-life installations of this market setup have already been put in place. Their examples include Piclo, UK [152], Vanderbron [153], or SonnenCommunity [154].

Until today, the majority of research in that field has been focused on a p2p market setup that assumes that all peers are solely interested in minimising the cost of received energy or maximising the profits. However, some more conscious customers on the market may be interested not only in minimising the cost of the energy purchase, but, simultaneously, may have preferences on the energy source characteristics that include various quality criteria (the level of greenhouse gas emissions is the most obvious example, but there may be other preferences that may arise from various ecological and sustainability concerns, or social responsibility considerations).

Specifically, a peer may be interested in purchasing energy from particular types of *green* energy sources, or from producers that assure to comply *socially responsible* standards of their business. A real-life example of such interest is the commitment made by one of the world's leading technology consulting firms — Accenture. Accenture claimed that, by the year 2023, 100% of the energy it consumes worldwide will come from renewable energy sources (RES) [155]. This example shows that, nowadays, peers already exist that would like to consider various quality criteria, while making decisions on the origins of the consumed energy.

Although tempting, in the current market setups, it is not directly possible to guarantee that the energy consumed by a customer indeed comes from a particular type of the energy sources, since, once energy is injected into the grid, it is impossible to distinguish it at the destination. Today, one of the means to bypass this inconvenience is via the *tradable green certificates* (TGC) [156]. In this setup, an RES obtains a governmental certificate for the volume of energy it injected into the grid over a defined aggregated time horizon (year). After the certificates are issued, they are traded at dedicated markets. Therefore, a party that buys a certificate for a given volume of energy v_{cert} may claim that its consumed volume v_{cert} came from RES. However, this is a virtual commitment that lacks direct links to possible characteristics of the produced and consumed energy over shorter periods of time. Therefore, it is not easy for the majority of peers to follow and fully accept such arrangements. Moreover, the discussed peer should pay twice separately—once for the provision of the energy and then for the certificate itself.

In this Chapter, we analyse the market setup with peer-to-peer energy offerings controlled by the market operator, with explicit product differentiation, where origins of energy may be clearly identified, so that product characteristics can be understood by various actors (including households). We explicitly focus on the described environment. For this purpose, we apply the generic multi-criteria Services of General Interest dispatching optimisation problem with quality-based criteria (SDOQ). It is assumed that peers are not only entirely interested in minimising the cost of their energy, but may also be interested in attaining a given value of the quality criterion. For simplicity, we refer to non-cost criteria as *energy quality*. In the Chapter, we propose a way to introduce the additional quality criterion in a trusted manner. We use the reference point scalarisation method that seems to be well-suited to solve this multi-criteria optimisation problem. The resulting optimisation problem has to be solved by the responsible market operator, who acts as the SGI dispatcher (using terminology from previous Chapters).

In that sense, we develop both a tool (in the form of a mathematical optimisation problem) and a framework for power system balancing in a peer-to-peer (with Operator) market setup so that ecological, social, or sustainability criteria may be considered in addition to the cost criterion. It would address the willingness of some peers to buy from ecological or socially responsible sources. This development forms the contribution of this Chapter. When implemented, it could give possibilities to interested peers to buy energy by considering additional quality criteria, so it might imply the rise in demand for the services with higher values of these criteria, and would possibly provide better incentives for the development of desired generating technologies or sellers' business models.

The approach is tested on the IEEE 30-bus standard test system, over three different scenarios, where the impact of various actors/peers activities and different extensions are analysed.

5.2 Literature review

Peer-to-peer energy trading has been gaining interest for the past few years since more and more energy market participants are showing their willingness to participate directly in the market themselves, and many consumers are becoming prosumers [151].

A significant number of papers were published in the field. Some of them covered the idea of a general setup of such distributed markets, some of them proposed mathematical models of operating p2p markets and some looked at technical implications of various setups. Furthermore, some technical reports were already published, looking at the technical implications of p2p implementations [157]. It is worth noting that, apart from classic research papers, some review articles on existing methods for peer-to-peer and transactive energy paradigms have already been published [158, 159, 160, 161, 162], which shows increasing interest in that field.

Some important examples of papers focusing on general setup may be [163, 151, 164, 165].

Parag and Sovacool [151] look at three market setups that may successfully integrate prosumers, with peer-to-peer being one of them. However, the authors are cautious that some changes must be made prior to its large-scale introduction. Seeing the p2p market as a possibility for incentivising prosumers is also in line with the findings of [166]. Pires Klein et al. [163] developed a dedicated p2p business model, which was successfully applied in three physical, real-life, pilot projects, under Portuguese real energy market data. They demonstrated that such a model brings cost savings to participants and is indeed feasible to be implemented. However, they outlined that, due to the lack of precise legal regulations, it is not straightforward to be mounted. Baez-Gonzalez et al. [164] claim that peer-to-peer structures show benefits such as the ability to work in dynamically changing environments, scalability and symmetric role of peers, which are all important in power systems. However, they pinpoint some drawbacks too, such as concerns on security, fault tolerance, and the imperfect assumption that all peers are altruistic and share a common objective (*social welfare*). Zia et al. [165] look at a broader term, namely the *transactive energy paradigm* in microgrids. They propose a design for both peer-to-peer and community-based markets with its functional layers. One more example of analysis of the real-life peer-to-peer market is [167]. The authors analysed user behaviour in a real-life p2p market consisting of 37 households. They found that household users showed high interest in the energy market operations of the studied setup. They have also outlined that p2p may increase the saliency of renewable energy sources and may promote load shifting behaviours. However, not every location or currently existing microgrid may be considered a good candidate for establishing peer-to-peer trading mechanisms. The authors of [168] propose a method based on optimisation to assess the feasibility of setting up the p2p mechanism for a given microgrid.

From another perspective, we can pinpoint many papers focusing on producing appropriate mathematical models of peer-to-peer operation. The authors of [169] used the game theory simulation approach to show that the p2p trading scheme may improve the local balance of energy of a low-voltage microgrid. Other game theory approach examples are [170], whose authors analyse implications of p2p from an equilibria perspective and [171], where the authors propose a motivational psychology framework that may be used to design p2p markets.

Apart from game-theoretical approaches, much research interest is given to (distributed) optimisation models for peer-to-peer energy trading operations. The authors of [172] propose a mixed-integer nonlinear program for optimising operations of smart homes in a p2p environment. Due to the high computational burden, a heuristic for solving a similar problem is later proposed in [173]. The authors of [174] propose an optimisation for p2p decentralised operations, when considering network constraints. Nizami et al. [175] propose a model for prosumers equipped with an energy storage. Another approach is presented in [176], where electric vehicles (EVs) are explicitly considered in the model. Another paper worth noting is [177], where the authors integrate aggregators in power balancing over a transactive energy paradigm.

From the perspective of our research, however, one of the most important notions is *electricity product differentiation*, since in this work we are explicitly interested in a situation, where peers aspire for a given amount of *good* energy consumed. By *good*, we mean energy with quality characteristics that interest the peer — e.g., energy from an ecological source, or energy produced in a socially responsible manner. Therefore, the differentiation of its origin is important. The notion of energy differentiation in energy markets is described well in [178]. This concept has already been applied to peer-to-peer trading in [179, 180, 170]. However, the approach of all authors of these papers is to introduce peers' trading preferences (which is implied by the notion of product differentiation) as an additional cost component inserted implicitly into their respective cost functions. We believe that such an approach is of little tangibility to the peers and corresponding values may not be well understood by them. Furthermore, it may be difficult for consuming peers to correctly quantify their preferences in terms of the costs of trade with other actors. What is more, this cost component represents the bilateral preference that a given consumer j has on buying from a given seller s. Such a representation is very subjective and in some cases may compromise the notion of fair operations of energy markets.

All things considered, peer-to-peer energy trading is a subject of extensive research. However, few papers consider differentiation of the energy, and, to the best of our knowledge, none treats product differentiation as separate criteria in the multi-criteria optimisation framework. Moreover, despite their research value, the cited papers that consider product differentiation do not analyse the impacts and consequences for different actors. They also do not take into account that the approach could be interactive. Our Chapter intends to fill this gap. In that sense, we develop a mathematical optimisation tool, together with a market framework to operate the power system balancing with preference criteria additional to the cost of energy. When implemented, this tool may give possibilities for interested peers to buy energy from preferable sources and to imply the rise in demand for the services with higher values of these quality criteria. As a result, improvements in generating technologies or sellers' business models could be experienced.

5.3 Proposed balancing architecture

It is impossible to physically distinguish energy from different sources when already injected into the grid. Thus, we believe that only when energy is directly purchased by customers from particular sellers is it possible to truly reflect buyers' preferences on the origins of the purchased energy. Other known market mechanisms (such as, for instance, the trade of TGCs), although correctly reflecting the amount of physical energy injected, make it less tangible for customers to understand the origins of its purchased energy well.

In this work, we propose a balancing architecture for day-ahead over a horizon $h = 1, 2, \ldots, H$ accomplished in a peer-to-peer (*p2p*) manner. In the proposed architecture, each seller s submits individually priced offers to each buyer j over the considered balancing horizon. A detailed offering process is described further in Section 5.3.3.

We specifically consider not only the energy cost criterion but also other various ecological social or sustainability criteria that may be of interest to peers participating in the market. A detailed description of the mechanism for considering these criteria is given further in Section 5.3.4.

5.3.1 Role of Operator

It is important to remember that it is required for power systems to be operated safely and securely at all times. The energy/power balance in the power system must be met in each time period, subject to satisfying many technical and security constraints. In a truly distributed p2p environment, sellers submit crafted offers to dedicated buyers, and trade happens directly between peers. Generally, one may envisage the following problems:

- some consumers may receive offers that are not covering their demands;
- some peers may try to execute market power over others;
- dispatch of the generating units that result from bilateral trade may give network-infeasible power flows.

As a result, the contracted positions of all peers resulting from bilateral trade may appear to be infeasible. In order to resolve the above infeasibility issues, these balancing problems may be sorted out by the System Operator. The Operator may gather all balancing offers issued from sellers and buyers, check them for issues mentioned above, and reschedule all contract positions of all peers, by solving appropriate balancing problem that takes into account individual aspirations and reservations of each individual peer towards their all criteria.

Such an approach is viable, since the Operator may have knowledge of technical details and security issues of the system under operation. The Operator could serve as the *trusted third party* for peers — the SGI dispatcher. For simplicity of further considerations and notation, but without any loss of generality, we may assume that the contracted positions prior to balancing are set to zero and balancing offers can be simply treated as offers.

5.3.2 Proposed architecture

As already stated, we propose in this Chapter a power system balancing architecture/energy market framework. It can be simply summarised in the following points:

1. Each peer submits its aspirations and reservations towards all criteria present on the market, together with information on its technical constraints.

2. Sellers submit technically identical offers to buyers, but with different individual prices, as per (5.2). Prices are offered as per individual business decisions of sellers.

3. The copy of each offer is submitted to the Operator for review and joint optimisation.

4. Optimisation is performed by the Operator and results are returned to all peers.

5. System operates as per balancing dispatch calculated through optimisation.

A simplified version of the process described above is visualised in Figure 5.1. The figure illustrates a simple market with only one seller and two buyers.

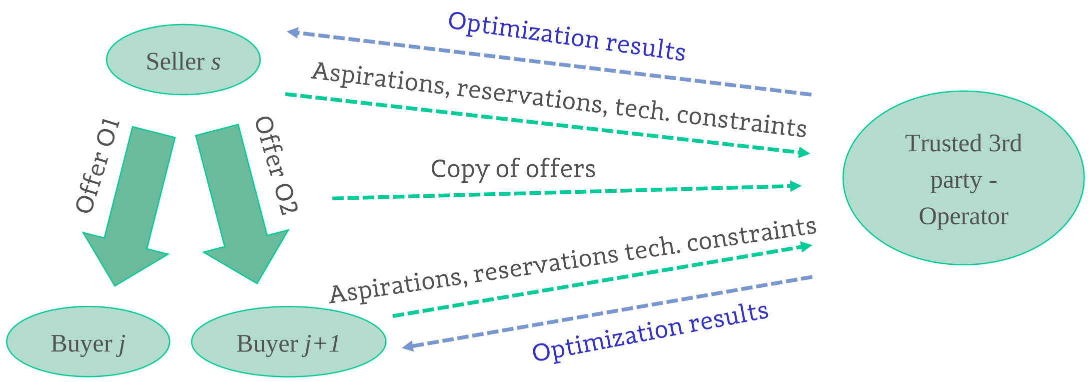


Figure 5.1: Schema of proposed balancing architecture.

5.3.3 Offers

5.3.4 Integration of additional criteria

As already described in the previous Sections of this Chapter, the cost of energy is not the only criterion for peers, as they may be interested in purchasing energy from particular *green* sources, or from producers that respect various ecological, social and/or sustainability concerns. Let us assume that these additional criteria (e.g., the amount of green energy produced, the ratio of social responsibility, or others) are quantifiable and linked directly to a given seller s (assumption forming the basis of the proposed market framework). Then, for s, we can denote them by vector $\mathbf{q}_s = [q_1, q_2, \ldots, q_m]$, where m is the number of all possible criteria taken into account and q_k is *certified* value for criterion k. For the proposed scheme to work properly, peers must be confident that submitted values of $\mathbf{q}_s$ are genuine. Therefore, in this work, we assume that there should exist a trusted *notified body* that would officially certify sellers on $\mathbf{q}_s$. This role may be held by the Operator. In the proposed scheme, the values of $\mathbf{q}_s$ are known to peers, whenever offers are submitted by s.

5.3.5 Multi-commodity offering mechanism

Green and/or socially responsible energy is often produced from renewable, highly variable sources. If the dispatch of units results from a single-period load optimisation performed by the Operator, a given source g may be dispatched with maximum available output for just one single period of time t and not be dispatched for other periods at all. From the producer's perspective, such a situation may be too costly or even infeasible — firstly because, for some units, its startup may be simply too expensive or infeasible; secondly, because some generating units are variable and often depend on the weather. In a case where, as a result of optimisation, a seller is committed for only a single time period, and due to force majeure happening in that very time period, it cannot deliver the demanded amount of energy, the seller may be charged high penalty costs. Therefore, it is desirable to balance generation with loads by considering some form of security-constrained unit commitment and economic dispatch problem over a longer horizon, say, consisting of many time periods $t = 1, 2, \ldots, H$.

A convenient way of handling multi-period requirements on the market is by using integrated offers that may represent bundles of commodities in a single offer, in order to clear up simultaneously many commodities on the *multi-commodity market*, as presented in [47]. Here, we can use this general approach to create multi-period offers that represent profiles of generated or consumed energy over time, in a joint manner for multiple time periods, but at a single, averaged unit price. If a seller s is able to sell an energy profile over multiple time periods, it can mitigate the risk of paying higher penalties than the total income over the balancing horizon.

It is worth noting that a given seller normally would be more interested in receiving the highest income over the entire balancing horizon, and not for a particular time period only. Thus, offering in a joint manner (for multiple time periods, at an averaged constant unit price) may bring additional benefits, while mitigating the risks of having the generating source dispatched for only one time period. However, as described further in this Section, when a multi-commodity offering mechanism is available on the market, it is only an option, and it does not restrain sellers from submitting many single-period offers with time-varying prices (it is always a matter of the individual seller's business decisions, how they wish to design their offers).

A single multi-commodity offer covers many time periods within the balancing horizon. The i^{th} offer of seller s is defined by a vector of parameters $\boldsymbol{\alpha}_{i,s} = [\alpha^1_{i,s}, \alpha^2_{i,s}, ..., \alpha^H_{i,s}]$, $0 \leq \alpha^t_{i,s} \leq 1 \; \forall t = 1, 2, ..., H$, $\sum_{t=1}^{H} \alpha^t_{i,s} = 1$, maximum volume $\bar{P}_{i,s}$ offered over entire horizon, and constant unit price $e_{i,s}$ (over the entire horizon), and the quality vector $\mathbf{q}_s$ described previously. Parameter $\alpha^t_{i,s}$ reflects the portion of offered volume $P_{i,s}$, $0 \leq P_{i,s} \leq \bar{P}_{i,s}$, in a given time period t. Hence, the amount of energy $P^t_{i,s}$ offered by s in time period t through offer i is given by (5.1)

$$P^t_{i,s} = \alpha^t_{i,s} \, P_{i,s}. \tag{5.1}$$

To conclude, a given multi-commodity offer may be formally described by the quadruple given in (5.2).

$$(\bar{P}_{i,s}, e_{i,s}, \boldsymbol{\alpha}_{i,s}, \mathbf{q}_s). \tag{5.2}$$

It is worth noting that the multi-commodity notation is general enough to also express standard single-commodity (single-period) offers, as a particular case. Table 5.1 gives examples of vector $\boldsymbol{\alpha}_{i,s}$ for a single-commodity and for a multi-commodity offer over balancing horizon $H = 3$. Assigning α_i^1 to 1 and all other $\alpha_i^{2,3}$ to 0 expresses a single-period offer, with a time-varying price. Therefore, the proposed notation allows for consideration of both time-varying and time-constant offering prices directly.

Table 5.1: Examples of $\boldsymbol{\alpha}_{i,s}$for single-commodity and multi-commodity offers.

	Single-commodity	Multi-commodity
α_i^1	1	0.4
α_i^2	0	0.5
α_i^3	0	0.1

5.4 Mathematical modelling and possible peers

This Section gives the mathematical formulation of the multi-criteria (mixed-integer) linear programming optimisation problem for balancing the energy system that takes into account the aspirations and reservations of peers towards all considered criteria. We describe separately exemplary peers that may take part in the proposed architecture with their corresponding mathematical models.

However, as already stated in Section 5.3.2, we propose that all balancing calculations are performed by the Operator. Therefore, the complete optimisation problem presented in Section 5.4.5 must be solved by the Operator. The individual optimisation subproblems of different peers are presented in this Section for the sake of deriving constraints to the problem of the Operator.

5.4.1 Producer g

In this work, we assume that all producers are formally certified on $\mathbf{q}_s$, and the origin of resources or fuels to produce energy is not considered. Therefore, a given producer g considers only one criterion, i.e., maximisation of its income. If we assume for simplicity (and without loss of generality) that the production cost function is constant, it is equivalent to profit maximisation. This having been said, the optimisation problem of g takes on the form of (5.3):

$$
\begin{aligned}
\max \quad & c_g = \sum_{i \in \mathcal{C}_g} (e_{i,g} \sum_{t=1}^{H} P_{i,g} \alpha_{i,g}^t) \\
\text{s.t.} \quad & \underline{P_t} \leq \sum_{i \in \mathcal{C}_g} P_{i,g} \alpha_{i,g}^t \leq \overline{P_t} \qquad \forall t = 1, 2, \ldots, H
\end{aligned}
\tag{5.3}
$$

where:

- $\underline{P_t}$ / $\overline{P_t}$ — min/max generating capabilities of g in time instance t (parameters);
- $\mathcal{C}_g$ — set of peers buying from g;
- description of the offer as in (5.2).

The constraint assures that for each time instance, the producer's sold energy stays within its technical generation capabilities.

5.4.2 Consumer i

In this work, we consider a situation where consumers are not only interested purely in minimising the cost of consumed energy but also in reaching their aspirations towards additional criteria **q**.

For simplicity and without losing the generality of derivations, in the remainder of this Chapter, we will be referring to a single additional criterion q present on the market and consequently to a single certified value of this criterion for sellers — q_s. Under the assumption that only one additional criterion is considered, a given consumer i is interested both in minimising the cost of its energy and in reaching its aspiration towards this additional criterion q. Assuming that i is willing to maximise q and minimise cost, i's optimisation problem is given in (5.4):

$$
\begin{aligned}
\max \quad & [-c_1^i, q_2^i] \\
\text{s.t.} \quad & c_1^i = \sum_{s \in \mathcal{K}_i} e_{i,s} \sum_{t=1}^{H} P_{i,s} \alpha_{i,s}^t, \\
& q_2^i = \frac{1}{\sum_{t=1}^{H} \Delta_t} \sum_{s \in \mathcal{K}_i} q_s \sum_{t=1}^{H} P_{i,s} \alpha_{i,s}^t, \\
& \sum_{s \in \mathcal{K}_i} P_{i,s} \alpha_{i,s}^t = \Delta_t \qquad \forall t = 1, 2, \ldots, H, \\
& P_{i,s} \geq 0 \qquad \forall j \in \mathcal{K}_i
\end{aligned}
\tag{5.4}
$$

where:

- c_1^i — total cost of energy bought by i (variable);
- q_2^i — q of i averaged over the entire balancing horizon (variable);

- $\mathcal{K}_i$ — set of selling peers that submitted an offer to i;
- Δ_t — i's demand for energy in a given time period t (parameter);
- description of the offer as in (5.2).

The third constraint assures that the demand of customer i is covered in each time instance, and the fourth constraint is the mapping of variables related to the sellers with the ones related to i.

5.4.3 Broker b

In the described market setup, we may also foresee the participation of brokers. Their role may be to group small buyers and represent them on the peer-to-peer market. Brokers would buy energy on their behalf and resell it to them afterwards. In that way, weak buyers may achieve better prices, in comparison to direct negotiations with producers. This is obviously due to the fact that the aggregated demand of broker b is usually much higher than the demand of single smaller buyers and in that way, b may negotiate much better unit prices. Furthermore, broker b may also serve to disaggregate multi-commodity offers into single-commodity ones or vice versa. We assume that a broker is a party that cannot store energy and needs to resell it immediately after it has been bought — i.e., in the same time period t.

A broker both buys and sells energy. On one hand, it is willing to reach aspirations towards q of buyers to whom it is selling the energy; on the other, as it sells the energy to buyers, b should equally be certified towards q as a seller. We propose that it is certified basing on a broker's reservation on q submitted to the Operator (averaged over the entire horizon). This is given in (5.5):

$$q_b = w\; r_b^q \tag{5.5}$$

where:

- w — empirically determined coefficient ($w \in [0; 1]$), describing the ratio of what percentage of the reservation has been finally determined by optimisation, usually $w \approx 1$
- r_b^q — reservation of b towards q over the entire balancing horizon.

A broker is willing to fulfil the aspirations/reservations of represented buyers. Thus, it looks at two criteria — profit maximisation and reaching the desired value of q. The broker maximises its profit (not income) as the cost components are well known to the Operator and the broker itself. Since the value of criterion q is directly linked to the amount of energy bought over the balancing horizon, it is necessary that this amount is estimated.

Due to the high variability of flows to and from the broker, we propose to estimate the volume of energy bought by b on the basis of forecasts of the sales. Hence, the expected values can be used in the optimisation model. We make this assumption as it is a commonly used method by many Distribution System Operators to estimate energy demand profiles of households basing on typical (expected) consumption, as documented in one of the Polish DSO's (*Tauron Dystrybucja*) Distribution Grid Code [181].

Having said all of the above, the broker's optimisation problem takes the form of (5.6):

$$
\begin{aligned}
\max \quad & [c_1^b, q_2^b] \\
\text{s.t.} \quad & c_1^b = \sum_{m \in \mathcal{C}_b} e_m \sum_{t=1}^{H} P_m \alpha_m^t - \sum_{l \in \mathcal{K}_b} e_l \sum_{t=1}^{H} P_l \alpha_l^t, \\
& q_2^b = \frac{1}{\mathbf{E}(\sum_{l \in \mathcal{K}_b} P_l)} \sum_{l \in \mathcal{K}_b} q_l P_l, \\
& \sum_{m \in \mathcal{C}_b} P_m \alpha_m^t = \sum_{l \in \mathcal{K}_b} P_l \alpha_l^t \qquad \forall t = 1, 2, \ldots, H, \\
& P_l, P_m \geq 0 \qquad \forall i \in \mathcal{C}_b, l \in \mathcal{K}_b
\end{aligned}
\tag{5.6}
$$

where:

- c_1^b — broker's total profit (variable);
- q_2^b — broker's value of q averaged over entire horizon (variable);
- $\mathcal{K}_b$ set of selling peers that submitted offers to b;
- $\mathcal{C}_b$ set of buying peers to whom b submits its offers;
- $\mathbf{E}(\sum_{l \in \mathcal{K}_b} P_l)$ — expected amount of energy bought by b during the entire exchange horizon (parameter);
- description of the offer as in (5.2).

The third constraint assures that for each time instance, the amount of energy sold by the broker equals the energy bought by the broker. This is in line with our assumption that the broker has no ability to store energy and must re-sell it immediately after buying. The fourth constraint assures mapping between problems of buyers, sellers and brokers.

5.4.4 Flexible Prosumer with Storage (FLECSP) f

In the peer-to-peer market environment, we can foresee the existence of the most general type of peers, flexible prosumers with storage, denoted as *FLECSP* in this work. They are able to act simultaneously as producers, consumers, and storage operators. They may then produce energy for their own needs, sell its excess to interested buyers, and use storage to shift time periods of different actions (e.g., buy energy in time period t and sell it in $t + 5$).

In that way, FLECSP should be certified on q equally as a seller. However, it possesses both its own generating source and can buy energy from others to resell it to other peers afterwards. Thus, its certification on q should take into consideration both its own source and the fact of buying energy from others. We propose to denote it as in (5.7):

$$q_f = \frac{q_f^g + q_f^m}{2} \tag{5.7}$$

$$q_f^m = w\, r_f$$

where:

- r_f—reservation of f on q of energy being *bought*;
- q_f^g—officially certified value of q of FLECSP's generating unit;
- w—empirically determined coefficient ($w \in [0, 1]$), describing the ratio of what percentage of the reservation has been finally determined by optimisation, usually $w \approx 1$.

With such defined certification, we can write FLECSP's optimisation problem, as in (5.8). Similarly, as for the broker b, FLECSP is interested both in maximising its profit and in reaching the targeted level of q over the balancing horizon. Charging and discharging of storage units cannot happen simultaneously. This is modelled using binary variables, causing the optimisation problem to be a mixed-integer program (MIP):

$$
\begin{aligned}
\max \quad & [c_1^f, q_2^f] \\
\text{s.t.} \quad & c_1^f = \sum_{m\in\mathcal{C}_f} e_m \sum_{t=1}^{H} P_m \alpha_m^t - \sum_{l\in\mathcal{K}_f} e_l \sum_{t=1}^{H} P_l \alpha_l^t \ , \\
& q_2^f = \frac{1}{\sum_{t=1}^{H} \Delta_t^f} \big(\sum_{l\in\mathcal{K}_f} q_l P_l + P_{fg1}^t \, q_f + \sum_{t=1}^{H} P_{ffs1}^t - \sum_{t=1}^{H} P_{ffs}^t\big) \ , \\
& P_{ffs}^t + \sum_{l\in\mathcal{K}_f} P_l \alpha_l^t + P_{fg}^t = P_{fts}^t + \sum_{m\in\mathcal{C}_f} P_m \alpha_m^t + \Delta_t^f \qquad \forall t = 1, 2, \ldots, H, \\
& \Delta_t^f + P_{fts}^t = P_{ffs1}^t + P_{fg1}^t + \sum_{l\in\mathcal{K}_f} P_l \alpha_l^t \qquad \forall t = 1, 2, \ldots, H, \\
& \sum_{m\in\mathcal{C}_f} P_m \alpha_m^t = P_{ffs2}^t + P_{fg2}^t \qquad \forall t = 1, 2, \ldots, H, \\
& \Delta_t^f \le P_{ffs}^t + \sum_{l\in\mathcal{K}_f} P_l \alpha_l^t + P_{fg}^t \qquad \forall t = 1, 2, \ldots, H, \\
& P_{ffs1}^t + P_{ffs2}^t = P_{ffs}^t \qquad \forall t = 1, 2, \ldots, H, \\
& P_{fg1}^t + P_{fg2}^t = P_{fg}^t \qquad \forall t = 1, 2, \ldots, H, \\
& \underline{P_{fg}^t} \le P_{fg}^t \le \overline{P_{fg}^t} \qquad \forall t = 1, 2, \ldots, H, \\
& 0 \le P_{ffs}^t \le SOC^t \qquad \forall t = 1, 2, \ldots, H, \\
& SOC^t = SOC^{t_1} + \eta P_{fts}^{t-1} - \eta P_{ffs}^{t-1} \qquad \forall t = 1, 2, \ldots, H, \\
& \underline{SOC} \le SOC^t \le \overline{SOC} \qquad \forall t = 1, 2, \ldots, H, \\
& 0 \le P_{fts}^t \le M \, y_1^t \qquad \forall t = 1, 2, \ldots, H, \\
& 0 \le P_{ffs}^t \le M \, y_2^t \qquad \forall t = 1, 2, \ldots, H, \\
& y_1^t + y_2^t \le 1 \qquad \forall t = 1, 2, \ldots, H, \\
& y_1^t, y_2^t \in \{0, 1\} \qquad \forall t = 1, 2, \ldots, H, \\
& P_l, P_m \ge 0 \qquad \forall i \in \mathcal{C}_f, l \in \mathcal{K}_f
\end{aligned}
\tag{5.8}
$$

where:

- c_1^f — FLECSP's total profit (variable);
- q_2^f — FLECSP's value of q averaged over entire horizon (variable);
- $\mathcal{K}_f$ — set of selling peers that submitted offers to f, in the general situation f is also included in this set as it can produce energy for its own needs;
- $\mathcal{C}_f$ — set of buying peers to whom f submits its offers in the general situation f is also included in this set;

- P^t_{ffs} — total amount of energy taken from storage in time period t (variable);
- P^t_{fts} — amount of energy sent to storage in t (variable);
- P^t_{fg} — total amount of energy generated in t (variable);
- Δ^f_t — amount of energy demanded for f's needs in t (parameter);
- P^t_{ffs1}, P^t_{ffs2} — amount of energy taken from storage for f's internal needs and for sale to others in t respectively (variables);
- P^t_{fg1}, P^t_{fg2} — amount of energy generated by f for f's internal needs and for sale to others in t respectively (variables);
- SOC^t — state-of-charge of the storage in t (variable);
- $\underline{SOC}/\overline{SOC}$ — min/max limits of SOC (parameters);
- η — effectiveness of charging/discharging system (parameter);
- $\underline{P^t_{fg}}/\overline{P^t_{fg}}$ — min/max capabilities of f's generating source in t (parameters);
- M — constant large enough not to limit variables (parameter);
- y^t_1, y^t_2 — binary variables for storage modelling;
- description of the offer as in (5.2).

Constraints 3 to 8 assure the FLECSP's power balance, distinguishing whether a given amount of energy was taken for FLECSP's needs or for sale to others. Constraints 9 and 10 assure that the amounts of energy generated by FLECSP or taken from storage are within their limits. Constraints 11 and 12 are model the charging phenomenon of f's storage, considering its effectiveness. Constraints 13 and 14 are the Big-M ones to allow for constraint 15 to be written, which assures that charging and discharging of the storage cannot happen simultaneously. Constraint 17 assures for the mapping of variables between different actors.

One should notice that the formulation derived does not allow for the FLECSP to act as a broker, meaning that buying energy from other peers and then re-selling it is not possible. This is because we want the FLECSP to be as close to a regular prosumer as possible and because a dedicated broker peer is already modelled separately. If one decides, that broker activities are needed, then the model can be extended accordingly. This can be easily achieved by dividing the model of the offered received by f from others into two parts — one for re-sell, and the other for internal needs. This is however deemed out of scope for this work.

5.4.5 Operator

As stated previously in Section 5.3.2, the only complete optimisation problem considered in this work is the one for the Operator. It is considered to be the dispatcher of this specific SGI (energy generation). All previously described mathematical models, i.e., the ones of the producers, consumers, brokers and FLECSPs were prevented only to formulate the Operator's optimisation problem. All other actors' sub-problems are built into the Operator's problem in the form of constraints.

Having said this, we propose the Operator's optimisation problem in the form of (5.9). Please note, that the vector of criteria consists of all criteria, for all participants. Due to spacing limitations, we write it in a compound form.

$$\begin{aligned}
\text{max} \quad & [c_g, -c_1^i, q_2^i, c_1^b, q_2^b, c_1^f, q_2^f] \qquad \forall g \in \mathcal{G}, i \in \mathcal{I}_c, b \in \mathcal{B}, f \in \mathcal{F} \\
\text{s.t.} \quad & Producers'\ constr. \qquad \forall g \in \mathcal{G}, \\
& Consumers'\ constr. \qquad \forall i \in \mathcal{I}_c, \\
& Brokers'\ constr \qquad \forall b \in \mathcal{B}, \\
& FLECSPs'\ constr. \qquad \forall f \in \mathcal{F}, \\
& + DC - OPF\ network\ constr.\ \text{(optionally)}
\end{aligned} \tag{5.9}$$

where:

- $\mathcal{G}$ — set of producers (suppliers),
- $\mathcal{I}_c$ — set of consumers (customers),
- $\mathcal{B}$ — set of brokers,
- $\mathcal{F}$ — set of FLECSPs.

5.4.6 Incorporating enhancements towards network feasibility

The power flow can only be technically attainable (in this Chapter referred to as *network-feasible*) if all the variables, i.e., generating units setpoints, branch power flows, nodal voltages and nodal voltage angles fall within their technical limits. As it was shown in [182], the network feasibility of the power flow, considering both active and reactive flow, depends highly on the grid model used for determining the power dispatch of generating units. Thus, depending on the model of dispatching the units, despite the fact that all generation setpoints lay within generating units' capabilities, a non-feasible power flow may be obtained [20].

Therefore, additionally to the already described constraints, we optionally add to (5.9) the network constraints of the standard linear DC Optimal Power Flow problem in the multi-period form. DC-OPF is a linear approximation of the AC-OPF problem, for modelling active power flows within the power grid in the optimisation problem. Inclusion of the network constraints to the proposed optimisation problem allows the Operator to determine a dispatch of the generating units which is more likely to be network-feasible or be closer to feasibility. One should remember that it is still an approximation and does not fully guarantee meeting all constraints.

Since the DC-OPF problem is well known in the literature and does not form the main scope of this work, formulation of those constraints is omitted in this Chapter. However, an interested reader may consult references [183, 184, 185, 186] for detailed explanations.

Furthermore, to assure for even more secure transmission, for each time period, the Operator (together with negotiating peers) may iteratively run the method described in [20]. However, this is given only as a possible extension to the proposed framework and is not studied within this Chapter.

5.5 Relation with SGI dipatching problem and scalarisation

As already stated, the only problem which is solved in operating the energy market considered is the one of the Operator (5.9), understood as the SGI dispactcher. It is derived as a specific implementation of the generic SGI dispatch optimisation problem SDOQ (3.1). One may notice, that in (5.9) we have two criteria per each customer — one quality criterion and one cost criterion, and one per supplier — income (or profit). Those are given as specific definitions of the generic functions $f^1(\boldsymbol{x})$ and $f^2(\boldsymbol{x})$ of the SDOQ problem (3.1). In this Case Study's specific setting, we also consider participants who are both customers and suppliers — namely brokers and FLECSPs. Since they are present also on the demand side, they carry both monetary and quality criteria. All the above criteria are put into the optimisation problem (5.9). Optimisation variables then reflect the amount of service (electrical energy) offered by suppliers to the customers. Here, the vector $\boldsymbol{x}$ is composed of continuous decision variables P_π, where π is the specific index for the supplier peer considered (producer, broker, FLECSP). Those variables are only two-dimensional as in this Case Study the time dimension is treated through the α_π^t parameter. The generically described feasible set in the SDOQ problem — $\mathcal{Q}$, in this Case Study is defined by means of auxiliary variables specific to the energy generation problem together with many case-specific constraints. The constraints, which are specific to the operations of the peers are part of their respective models (5.3) - (5.8) and then transferred to the Operator's/dispatcher's problem. What is more, some constraints defining $\mathcal{Q}$, which are specific to the operations of the complete system (i.e., DC-OPF constraints) are built directly into the Operator's problem (5.9). The demand for the SGI service (given as Δ_i in SDOQ) is incorporated into the models of energy consumers and FLECSPs and results from their physical needs for electricity.

We select to scalarise the problem proposed (5.9) using the Reference Point Method scalarisation in the multi-criteria non-equitable formulation. This decision follows from the conclusions drawn from the analysis from the Chapters 3 and 4. We decide not to use the equitable formulation as, according to our findings summarised in Chapter 4, the solution time of this formulation is much higher than the non-equitable one. At the same time, any possible loss in performance of the functional indices is significantly lower than the gain in solution time. Here, one should remember that the dispatcher needs to make quick and accurate decisions on how to dispatch generating units.

Despite our decision to proceed with the non-equitable problem for this Case Study, in a real-life situation the Decision Maker may want to implement the equitable version of SDOQ. However, this should be a subject of a deep DM's cost-benefit evaluation prior to implementing.

5.6 Case Study

The proposed multi-criteria energy dispatch framework with quality-based criteria has been tested on the IEEE-30 standard 30-bus, 6-generator test system available [187]. The dispatch of the generators considered has been calculated using the problem (5.9) in various configurations. The one-line schematic is shown in Figure 5.2. Unfortunately, the test system does not have any network flow limits imposed. Thus, we take them from *Case 30* available in MATPOWER [188], which has been derived from the IEEE-30 system. Those network constraints are imposed only in Sec. 5.6.3. We decide to perform the Case Study on the standard IEEE 30-bus since it is a well-renowned standard system. For mathematical modelling of the approach, we used Matlab with MATPOWER and CVX — a package for specifying and solving convex programs [145, 146].

For all tests, we assume that the producer in node 5 is a photo-voltaic farm whose generation profile outcomes from real-life solar radiation data are taken from Pescara, Italy. Historical solar radiation data were obtained from SOLCAST [189]. We also assume that the producer in node 1 can produce a maximum of 100 MW in each time instance (i.e., $\overline{P_t} = 100$ MW).

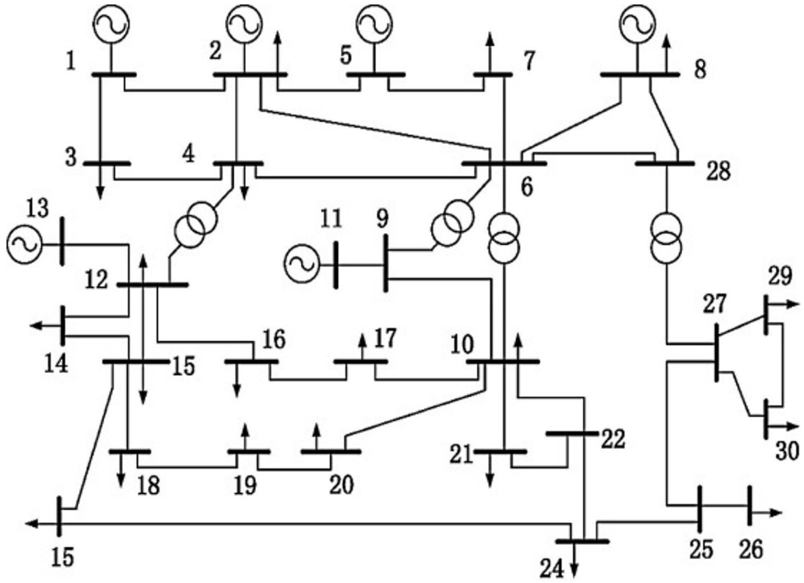


Figure 5.2: IEEE 30-bus system. Graphics taken from [190].

In the Case Study, we assume certifications on q_s of the generating units as given in Table 5.2. We assumed that higher certified values of q_s imply higher prices offered by generating units.

Table 5.2: Assumed values of q_s.

Gen ID	q_s
1	0.2221
2	0.4565
5	0.9000
8	0.7981
11	0.8919
13	0.8303

Tests have been performed in a few different settings, depending on the types of actors present and analysed. For the test, we assumed a day-ahead 15-min balancing horizon, implying that the optimisation horizon is equal to 96 time periods — i.e., $H = 96$. Demand changed from one time period to another such that a random component was added to the load attached to the bus as specified in the IEEE 30 test system available in MATPOWER.

First, we look at a simple market with producers and consumers only. Then, we also analyse the impact of the multi-commodity offers and inclusion of network constraints to (5.9). Furthermore, we also analyse the impact of introducing brokers and also FLECSP prosumers to the discussed setup. In the last cases, for simplicity, generally, we present results obtained when peers submitted single-commodity offers only. Operations on the multi-commodity mechanism are limited to Section 5.6.2 only.

Due to space limitations we do not cite the input data in this work. However, all data for all experiments are available in the online storage under the link: `https://bit.ly/3rPZATG`.

5.6.1 Simple market — producers and consumers

In this Section, we analyse a simple market structure, where only producers g and consumers i are present. We assume they are attached to buses as in Figure 5.2. We tested the approach over three scenarios, differing by values of aspirations and reservations of peers.

In Scenario 1, none of the consumer peers care about q, all of them want to pay as little as possible for their energy. On the other hand, all producers want to earn as much as possible. This is expressed by their aspirations a_i^{cost} and a_g^{income}, respectively. The interval between reservations (r_i^q) and aspirations (a_i^q) of consumers towards q is large enough to cover the entire space of possible values q — to reflect consumers' indifference towards this criterion.

In Scenario 2, consumer 9 wants to have a higher value of q associated with consumed energy, ideally equal to 0.9. This is expressed by its aspiration a_9^q towards this criterion. However, the peer would also accept $q = 0.8$ as its reservation towards this criterion, therefore $r_q^q = 0.8$. At the same time, other consumers would still want to pay as little as possible and only consumer 9 agrees to pay more for energy with a higher value of q.

In Scenario 3, however, the situation slightly changes. Here, we consider not only Consumer 9 as aspiring for $q = 0.9$, but also the remaining consumers wish to have values of q within interval $[0.5; 0.6]$. The assumed values of the reservations and aspirations for different scenarios are summarised in Table 5.3.

Table 5.3: Assumed values of aspirations and reservations taken for different scenarios.

	Scenario 1	Scenario 2	Scenario 3
a_g^{income} [\$]	5.00×10^5	5.00×10^5	5.00×10^5
$a_{i=1,2,..,21\setminus\{9\}}^{cost}$ [\$]	1.00×10^2	1.00×10^2	1.00×10^2
a_9^{cost} [\$]	1.00×10^2	1×10^2	1.00×10^2
$a_{i=1,2,..,21\setminus\{9\}}^q$	0.00	0.00	0.60
a_9^q	0.00	0.90	0.90
r_g^{income} [\$]	5.00×10^3	5.00×10^3	5.00×10^3
$r_{i=1,2,..,21\setminus\{9\}}^{cost}$ [\$]	4.00×10^4	4.00×10^4	5.00×10^4
r_9^{cost} [\$]	4.00×10^4	6.00×10^4	6.00×10^4
$r_{i=1,2,..,21\setminus\{9\}}^q$	1.00	0.00	0.50
r_9^q	1.00	0.80	0.80

In this test, only single-commodity offers were submitted. Unit prices were taken randomly, under the assumption that, for a higher value of q_s, the offering price is also higher.

We compare the results obtained by solving the problem (5.9) in scalarised with the use of RPM in the non-equitable form, with the following alternative approaches (5.10), (5.11) where $\mathcal{Q}$ represents the feasible set of the problem considered and $\boldsymbol{x}$ — vector of decision variables. The results obtained are shown in Tab. 5.4, where by (5.9) we mean the problem proposed scalarised with RPM in the non-equitable formulation.

$$\begin{aligned} \min \quad & \sum_{i \in \mathcal{I}_c} c_1^i \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{Q} \end{aligned} \tag{5.10}$$

$$\begin{aligned} \min \quad & v_1 \sum_{i \in \mathcal{I}_c} c_1^i - v_2 \sum_{g \in \mathcal{G}} c_g - v_3 \sum_{i \in \mathcal{I}_c} q_2^i \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{Q} \end{aligned} \tag{5.11}$$

where: v_1, v_2, v_3 — arbitrarily chosen positive weights.

In the comparisons, we look at the indices given in the points below. Due to space limitations, we do not cite the indices in full in the table but only give their respective item number (numbering as presented below). Similarly to the previous experiments, we assume that the aspirations and reservations reflect accurately participants' preferences, as they are submitted by themselves personally. We also check the values of quality received by consumers 9 and 1. We look at Consumer 9 since it is assumed to aspire for higher values of q in Scenarios 2 and 3. Consumer 3 is randomly taken for comparison. Numerical results of tests are presented in Table 5.4. Having said the above, the indices considered are:

1. Number of criteria being at least as good as their reservations associated (max. 48);
2. Number of criteria being worse from their reservations by at least 10%;
3. Maximum percentage gap between the criterion value and its reservation, for both cost and quality criteria jointly;
4. Mean value of the utility function;
5. Value of q_2^i received by Consumer 9;
6. Value of cost paid by Consumer 9 [$];
7. Value of q_2^i received by Consumer 3;
8. Value of cost paid by Consumer 3 [$].

The description and formulation of the indices are as given in Chapter 4. We omit here the investigations of the optimiser's solution time, as in the Case Study we focus more on the functional attributes of the solutions obtained with respect to operating the energy market. What is more, the relation between the solution time and the problem formulation has already been studied in Chapter 4.

Table 5.4: Results of numerical tests — simple market, Scenario 1

Results of numerical tests — simple market Scenario 1									
Index	(5.9)	(5.10)	(5.11) $v_1 = 1$, $v_2 = 1$, $v_3 = 1$	(5.11) $v_1 = 0.8$, $v_2 = 15$, $v_3 = 1.2$	(5.11) $v_1 = 0.8$, $v_2 = 1.2$, $v_3 = 15$	(5.11) $v_1 = 15$, $v_2 = 1.2$, $v_3 = 0.8$	(5.11) $v_1 = 15$, $v_2 = 0.8$, $v_3 = 1.2$	(5.11) $v_1 = 1.2$, $v_2 = 0.8$, $v_3 = 15$	(5.11) $v_1 = 1.2$, $v_2 = 15$, $v_3 = 0.8$
1	43.00	40.00	32.00	34.00	34.00	40.00	40.00	40.00	34.00
2	5.00	7.00	15.00	14.00	14.00	7.00	7.00	7.00	14.00
3	76.42%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
4	−112.52	−141.10	−532.46	−569.35	−569.31	−141.10	−141.10	−140.88	−569.35
5	0.54	0.40	0.89	0.85	0.85	0.40	0.40	0.90	0.85
6	2.27×10^4	1.63×10^4	4.46×10^4	3.87×10^4	3.87×10^4	1.63×10^4	1.63×10^4	1.90×10^4	3.87×10^4
7	0.63	0.37	0.89	0.85	0.85	0.37	0.37	0.37	0.85
8	3.15×10^4	1.90×10^4	5.49×10^4	4.72×10^4	4.72×10^4	1.90×10^4	1.90×10^4	1.63×10^4	4.72×10^4

One can note that even in Scenario 1 the multi-criteria approach using the RPM scalarisation allowed to produce results, which outperformed all the alternative approaches in the overall market indices studied, i.e., lowest maximum gap and the highest mean value of the utility and the lowest number of criteria worse by at least 10% than their reservation, and in the number of criteria being as good as their reservation. Thus, we may conclude that the proposed approach allowed us to produce results more tailored to customers' needs in Scenario 1.

Results obtained for Scenario 2 are shown in Tab 5.5. We see clearly from the results, that once again the proposed approach managed to produce more tailored results to participants' needs. This time it outperforms the alternative approaches in all the overall market indices studied. What is more, it also gives the quality results as requested by Consumer 9 — close to 0.9 with its cost in the acceptable limits. Thus, the behaviour is just as expected.

Table 5.5: Results of numerical tests — simple market, Scenario 2

Results of numerical tests — simple market Scenario 2									
Index	(5.9)	(5.10)	(5.11) $v_1 = 1$, $v_2 = 1$, $v_3 = 1$	(5.11) $v_1 = 0.8$, $v_2 = 15$, $v_3 = 1.2$	(5.11) $v_1 = 0.8$, $v_2 = 1.2$, $v_3 = 15$	(5.11) $v_1 = 15$, $v_2 = 1.2$, $v_3 = 0.8$	(5.11) $v_1 = 15$, $v_2 = 0.8$, $v_3 = 1.2$	(5.11) $v_1 = 1.2$, $v_2 = 0.8$, $v_3 = 15$	(5.11) $v_1 = 1.2$, $v_2 = 15$, $v_3 = 0.8$
1	43.00	39.00	33.00	34.00	34.00	39.00	39.00	39.00	34.00
2	5.00	8.00	14.00	14.00	14.00	8.00	8.00	8.00	14.00
3	76.42%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
4	−112.51	−223.66	−530.03	−569.35	−569.30	−223.66	−223.66	−223.45	−569.35
5	0.89	0.40	0.89	0.85	0.85	0.40	0.40	0.40	0.85
6	4.39×10^4	1.63×10^4	4.46×10^4	3.87×10^4	3.87×10^4	1.63×10^4	1.63×10^4	1.90×10^4	3.87×10^4
7	0.53	0.37	0.89	0.85	0.85	0.37	0.37	0.37	0.85
8	2.68×10^4	1.90×10^4	5.49×10^4	4.72×10^4	4.72×10^4	1.90×10^4	1.90×10^4	1.63×10^4	4.72×10^4

We now investigate the behaviour of the proposed problem over Scenario 3. The results are shown in Tab. 5.6. It is immediately noticeable that, again, the proposed approach allowed to produce results which are more tailored to participants' needs. In that sense, it allowed to yield results giving the majority of consumers the level of quality they demanded. The above conclusion may be drawn by looking at all overall market indices. Similarly to Scenario 2, the proposed approach outperformed all other alternative ones investigated.

It is worth noting that the problem proposed allowed us to both take into consideration the criteria (together with linked preferences) of both producers and consumers and to clear the market accordingly. It is an interesting quality, as it makes it possible to attract both the demand and supply side market participants.

Table 5.6: Results of numerical tests — simple market, Scenario 3

Results of numerical tests — simple market Scenario 3									
Index	(5.9)	(5.10)	(5.11) $v_1 = 1$, $v_2 = 1$, $v_3 = 1$	(5.11) $v_1 = 0.8$, $v_2 = 15$, $v_3 = 1.2$	(5.11) $v_1 = 0.8$, $v_2 = 1.2$, $v_3 = 15$	(5.11) $v_1 = 15$, $v_2 = 1.2$, $v_3 = 0.8$	(5.11) $v_1 = 15$, $v_2 = 0.8$, $v_3 = 1.2$	(5.11) $v_1 = 1.2$, $v_2 = 0.8$, $v_3 = 15$	(5.11) $v_1 = 1.2$, $v_2 = 15$, $v_3 = 0.8$
1	42.00	20.00	34.00	35.00	35.00	20.00	20.00	20.00	35.00
2	6.00	28.00	13.00	11.00	11.00	28.00	28.00	28.00	11.00
3	71.80%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
4	−143.44	−653.98	−372.63	−405.97	−405.93	−653.98	−653.98	−650.05	−405.97
5	0.89	0.40	0.89	0.85	0.85	0.40	0.40	0.40	0.85
6	4.38×10^4	1.63×10^4	4.46×10^4	3.87×10^4	3.87×10^4	1.63×10^4	1.63×10^4	1.63×10^4	3.87×10^4
7	0.60	0.37	0.89	0.85	0.85	0.37	0.37	0.37	0.85
8	2.99×10^4	1.90×10^4	5.49×10^4	4.72×10^4	4.72×10^4	1.90×10^4	1.90×10^4	1.90×10^4	4.72×10^4

The tests performed revealed that by considering quality-based criteria in the dispatch of generating units in the simple market, allowed us to produce results which respond better to participants' needs. The level of responsiveness to these needs is reflected by the utility function together with information on how the optimiser manages to allocate the dispatch so that the reservations are met.

5.6.2 Introduction of the multi-commodity offers

In this Section, we present simulation results for the case when a multi-commodity mechanism was introduced. To better outline why this specific mechanism may be of interest to peers, we assume that the only peer which is able to submit the multi-commodity offers is the PV producer located in node number 5.

We assume that the above producer submits both single and multi-commodity offers to all consuming peers. Single-commodity offers have exactly the same unit price as in previous experiments. However, the unit price for multi-commodity offers is lower due to the fact of a larger volume of offerings. It is then up to the Operator to Pareto-optimally decide on offers taken by solving (5.9). In our test, for simplicity, we assumed that the unit price in a multi-commodity offer is equal to $85.58\$$ to all consumers. This makes 80% of the average unit price offered through a single-commodity mechanism to all consumers.

For experiments, we take Scenario 3, as parameterised in Table 5.3. We specifically compare calculated profiles of generation of the PV producer in node 5 between situations where only single-commodity and both single and multi-commodity offers are submitted. Calculated PV generation profiles are shown in Figures 5.3 and 5.4. Numerical results are shown in Table 5.7. To keep consistency with previous experiments, we look at the same indices 1 to 8.

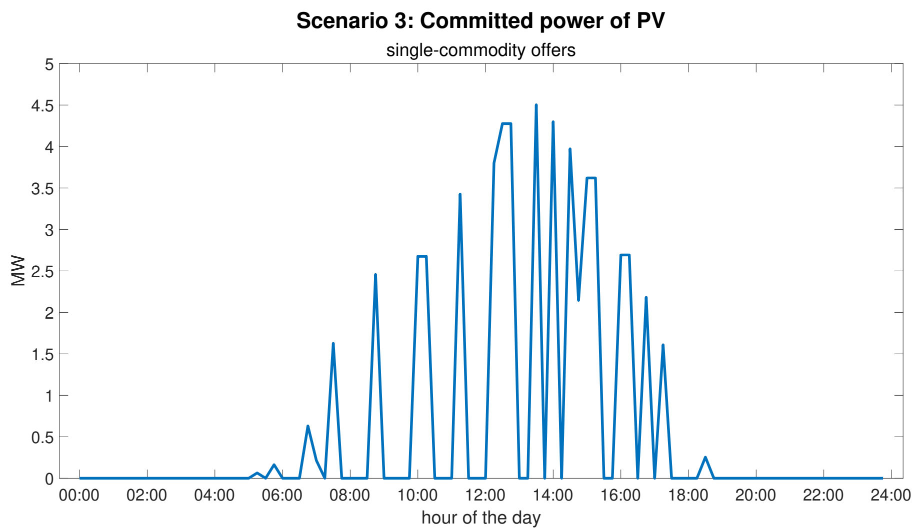


Figure 5.3: PV generation profile when only single-commodity offers are submitted.

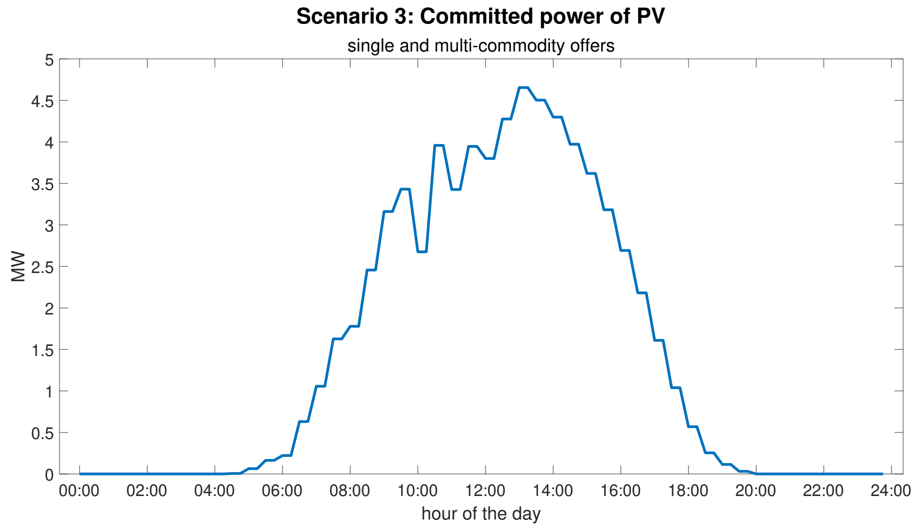


Figure 5.4: PV generation profile when both single and multi-commodity offers are submitted.

Table 5.7: Numerical results while having single-commodity offers only and single + multi-commodity.

Index	Single-commodity offers only	Single and multi-commodity offers
1	42.00	42.00
2	6.00	6.00
3	71.80%	71.80%
4	-143.45	-143.45
5	0.89	0.89
6	4.38×10^4	4.40×10^4
7	0.60	0.60
8	2.99×10^4	2.92×10^4

PV farm is a variable and dependent on weather energy source. The introduction of the multi-commodity offering mechanism allowed for committing the PV producer for the best possible generation profile over the whole time horizon rather than for a zigzagging output, as seen in Figure 5.3. This gives the possibility of reducing startup costs, as well as mitigating weather variation-related risks, without further restricting the feasible region of (5.9). When no special restrictions or no multi-commodity offers are in place, in an extreme situation, a variable source may be dispatched for one time period only. In case of a sudden weather change, the source concerned will not have any possibility of reacting to the new situation and will only be charged some penalties.

We can also see that the general overall market indices investigated (1 to 4) did not change, despite having the optimal solution changed — as can be seen by comparing the two production profiles depicted in Fig. 5.3 and in Fig. 5.4 and by comparing indices 6 and 8.

In that sense, we may conclude that the introduction of the multi-criteria mechanism may be successfully accomplished in the setup, where quality-based criteria are considered in addition to the cost ones. Such an introduction may allow for generally improving the balancing results (from an operational perspective) without compromising the fulfilment of peers' preferences. On the other hand, we have also shown that the energy balancing with the help of the SDOQ problem can also work in the setup where multi-commodity offers are considered. However, this is only given as a potential addition to the architecture and does not constitute the main body of this study.

5.6.3 Addition of network constraints

Ensuring that the power flow resulting from computed dispatch is technically feasible is of great importance to the Operator. Thus, we outline that it is possible to add to the problem (5.9) the standard DC-OPF network constraints. Although not necessary, having them in might improve the rate of network-feasibility of the optimised results.

In this Section, we present numerical results of the performed test, where we compared situations with and without the additional constraints under Scenario 3, with no multi-commodity offers allowed. We choose Scenario 3 for tests, as it is the most demanding one. To identify if a given dispatch is network-feasible or not, we solve the regular AC-OPF, as built in MATPOWER. We assume that the regular AC-OPF can change each unit's dispatch by no more than $+/-3\%$ of the output calculated by solving (5.9), for each time period. We permit these small changes to, for example, allow for the compensation of transmission losses, which are not estimated in the linear DC-OPF formulation. When under such a setup and such limitations, the AC-OPF converge — the power flow in the given time period is concluded to be network-feasible. Otherwise, it is concluded infeasible. Obtained results are presented in Table 5.8.

Table 5.8: Results of numerical tests — network constraints.

Index	DC-OPF not considered	DC-OPF Considered
1	42.00	42.00
2	6.00	6.00
3	71.80%	71.80%
4	-143.45	-143.45
5	0.89	0.88
6	4.40×10^4	4.33×10^4
7	0.60	0.60
8	2.91×10^4	3.00×10^4
Number of network-feasible time periods	79.00	90.00

As we see from the results, the number of feasible time periods increased with the addition of DC-OPF constraints under the Scenario investigated, yet still did not reach its maximum number. The increase in feasibility did not change the market indices studied (1 to 4). This shows, that the proposed approach can work also with the DC-OPF network constraints added, to improve the feasibility of the network flow.

5.6.4 Market with a broker

In this Section, we present some test results for the case when a broker is introduced to act on the market. In this case, balancing performance is tested under Scenario 3. We choose this Scenario since it is the most restrictive and therefore may better outline the differences in performance between cases with and without the broker.

For testing, we make some assumptions — as summarised in points below:

- one broker is introduced;
- the broker can buy from all producers, but can sell only to consumers in nodes 16, 18, 20, 23, and 29;
- since the broker represents many customers, it has special, lower prices negotiated - namely always 35$ per unit;
- the broker adds 30% of markup on the price;

- expected value of sold energy is equal to the total sum of demand of peers able to buy from the broker;
- the broker cannot store energy and must resell it in the same time period as bought;
- consumers are free to buy energy either from the broker or from the producers directly.

In this test, we assume that the broker's aspiration on profit — $a_b^{profit} = 5 \times 10^5$\$, with the reservation $r_b^{profit} = 1 \times 10^3$\$. The broker's aspiration and reservation on q (as a consumer) are equal to 0.61 and 0.6, respectively. This having been said, numerical results are shown in Table 5.9.

Table 5.9: Results of numerical tests — market with broker.

Index	No Broker	With Broker
1 (Broker's not considered)	42.00	42.00
2 (Broker's nor considered)	6.00	6.00
3 (Broker's not considered	71.80%	71.80%
4 (Broker's not considered)	-143.45	-140.52
5	0.89	0.89
6	4.40×10^4	4.35×10^4
7	0.60	0.60
8	2.91×10^4	3.12×10^4
Broker's profit [\$]	—	9.02×10^3
Broker's obtained value of q_2^b	—	0.63
Total cost paid by Broker's customers [\$]	5.71×10^4	5.43×10^4

As seen from the results, the proposed multi-criteria approach can work well also when a broker is introduced to the market. Such a broker may represent smaller, less powerful consumers by aggregating them. In that way, it may help in reducing the cost paid by its customers. The method also allowed for the broker to make the requested profit.

5.6.5 Market with a broker and with a FLECSP

In this Section, we analyse the case with both a broker and a FLECSP introduced to the market. We consider FLECSP to be a flexible prosumer equipped with a storage utility. It may produce energy for its own needs, sell to others and buy from others either to use for its own needs or to resell to other consumers. Since FLECSP is equipped with energy storage, it does not have to sell the energy right after buying/producing it but may shift time periods of delivery.

Similarly to a previous case with a broker only, for testing, we take the instance similar to Scenario 3 with single-commodity offers only. We make the below assumptions on FLECSP:

- FLECSP is located at node 2 of the studied network;
- FLECSP substitutes the producer located at node 2 in previous experiments;
- FLECSP has a 6.2 MW PV plant with 48 MWh storage;
- FLECSP offers its processed energy to all consuming peers on the market at the uniform price of 45$ per unit;
- Expected value of the energy processed by FLECSP is equal to node 2's total demand;
- FLECSP certified value of q_f^g equals 0.9, which for $w = 1$ and the below stated reservation gives $q_f = 0.7$;
- FLECSP's storage state-of-charge should be close to 0 at the end of the balancing horizon;
- FLECSP cannot act as a broker.

All assumptions on the broker's behaviour are identical to the ones in Section 5.6.4, with the exception of changed reservations towards quality as a consumer. This time aspiration on q equals 0.6 and reservation 0.5 — just as for other consumers.

Completely changed setup in the node 2 induced changing previously assumed pricing. What is more, we have substituted the largest generating unit, capable of producing 140 MW in each time instance. Therefore, comparing numerical results is of use only within this Section, and it is not relevant when comparing with previously shown results.

Numerical results with FLECSP in place are shown in Table 5.10. They are compared with the situation with no FLECSP installed. In this case, node 2 is a purely consuming node — with no generating unit nor storage capabilities at all. As such, obviously, it cannot trade on the market either. Assumed aspiration and reservation on the cost that node 2 needs to take are as follows—$a_{flecsp}^{cost} = 1 \times 10^2$\$ and $r_{flecsp}^{cost} = 9.0 \times 10^4$\$. However, values on q are 0.6 (aspiration) and 0.5 (reservation).

Table 5.10: Results of numerical tests—market with a broker and a FLECSP.

Index	No FLECSP capabilities	With FLECSP capabilities
1	43.00	44.00
2	5.00	5.00
3	71.94%	71.73%
4	-158.75	-156.74
5	0.84	0.85
6	3.54×10^4	3.72×10^4
7	0.81	0.84
8	4.00×10^4	4.30×10^4
Broker's profit [$]	8.46×10^3	7.68×10^3
Broker's obtained value of q_2^b	0.58	0.54
Node 2's cost [$]	1.06×10^5	9.00×10^4
Node 2's obtained value of q_2^f	0.74	0.59

As seen from the results, the introduction of FLECSP capabilities may improve the results of the consumer in node two due to the additional flexibility it provides. We have also shown in this Section that a more complicated way of modelling, where both a FLECSP and a broker are introduced, can work well in the proposed multi-criteria approach. Thus, one may conclude that the approach is general enough to accommodate multiple various types of peers, which may be present on the energy market.

5.7 Conclusions

In this Chapter, we present a Case Study of the application of the proposed SDOQ problem to handling electrical energy balancing with peer-to-peer offering.

We present the multi-criteria (mixed integer) linear program which is to be solved by the market Operator (acting as the SGI dispatcher), considering the aspirations and reservations of all peers present on the market towards their respective criteria. The idea behind the proposed approach is that all peers submit values of their aspirations and reservations towards their criteria to the Operator. Then, the Operator solves the proposed optimisation problem by finding Pareto-optimal solutions that respect submitted reservations and aspiration levels. Therefore, the resulting market positions reflect as much as possible the different preferences of the participating peers. Here, it should be noted that Pareto-optimality alone does not guarantee the maximisation of the total economical wealth or fairness/equity of the solutions. Yet, as already specified in the introduction to this Chapter, equitable optimisation is deliberately not considered due to significantly longer computational time, with not as significant a benefit linked to changing the solution.

The proposed optimisation problem is fairly general, so it allows for including multi - commodity peer-to-peer offers, network constraints, etc. The former significantly improves offering performance, especially for weather-dependent energy sources. In the Case Study, we observed that when multi-commodity offers are introduced, a photovoltaic source is committed for the complete capability profile during all time periods rather than for selected periods only. This happens without further constraining the feasible set of the problem. The inclusion of network constraints to the balancing problem restricts, however, the space of feasible solutions. However, we have shown that it may be helpful in obtaining technically feasible power flows in more instances.

For the proposed approach, we have identified many possible actors to be present within the discussed market setup. They include consumers, producers, brokers and flexible prosumers with storage. We have shown that such actors may be well integrated and modelled in the proposed approach, and each one of them may give some added value to the balancing.

In light of the two precedent paragraphs, we have showcased that the proposed approach, when applied to the energy market, can integrate multiple various peers and extensions to the problem. In that sense, it may work interestingly for real dispatching of the energy generating units. We have also shown that not only do we consider the interests of customers in paying low amounts, but also the interests of sellers in earning profits.

However, despite the fact that our tests were performed on the standard IEEE 30 test system, we studied the cases limited to actors described in the previous paragraph. We have not analysed other types of peers that may be envisaged. However, we believe that the set of peers reflects quite well a variety of possible interests. The described approach is purely conceptual, validated only in the Case Study given in Section 5.6 of the paper. Further simulation studies are desired, and a real-live prototype of the multi-objective market could be built to validate the approach under more realistic operating conditions.

This brings us smoothly to further research possibilities. As already stated, it might be interesting to build a pilot microgrid where the proposed setup could be further developed and tested. Second, the approach relies on multiple data exchanges between the peers and the Operator. The sensitive data should be kept private at all times, and therefore secure protocols of communication should be studied. Furthermore, the modelling of storage constraints of the flexible prosumer peer (FLECSP) is accomplished with the help of binary variables. For markets with many peers of this kind, the computational burden of the proposed mixed-integer linear program may be significantly increased. However, whenever found, the Pareto-optimality of the solution is guaranteed. To reduce the computational burden, some dedicated optimisation heuristics, or relaxation-based models, may be developed in future works.

It should also be noted that the proposed model considers directly the generation cost criterion only. However, one may also study the impact of other costs, such as the costs of obtaining formal certification of the quality attributes of the generating sources. Such a consideration would yield another interesting decision-making problem for sellers, namely to infer whether it is more beneficial to obtain higher certified values of q or not. However, this lies out of the scope of this paper as it is linked more to capital cost (CAPEX) considerations. However, we identify this problem as a topic of further research.

Despite the simplifications mentioned above, this Chapter shows both a framework and a tool to integrate the proposed generic multi-criteria Services of General Interest dispatching optimisation problem with quality-based criteria into the operations of energy markets. Such an optimisation tool, if implemented on the energy market, would allow interested peers to buy energy from sellers who are certified with higher values of the quality criterion so that it would increase the demand for that quality service. As a result of higher demand, technology shifts towards sources with higher quality (ecological, social, sustainability or others) may be experienced. In this Case Study, we have also shown that considering quality-based criteria in the dispatching of electrical energy generating units adds value to the dispatching and allows for yielding more tailored results to customers' needs.

5.8 Acknowledgements

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Chapter 6

Case Study II: Emergency Medical Services

In this Chapter, we put our proposed generic multi-criteria Services of General Interest dispatching optimisation problem with quality-based criteria (SDOQ) to action on operating the system of Emergency Medical Services (EMS) with Emergency Departments (EDs). The problem we consider is the Pareto-optimal assignment of ambulances to patients and then assignment of patients who are already on board the ambulances to appropriate EDs. This is done by considering both time-to-arrival and speciality (quality) of service criteria. We argue that the quality, further referred to as *speciality* of responding units (EMS or EDs), should be assigned as per the medical needs of the patients. In that way, their chances of survival in the emergency may be improved.

Similarly to the Case Study I, this Chapter is fully based on our published paper [63]. The majority of the text given in this Chapter is directly cited from this reference. It should be noted that the parts of the text describing the medical aspects of various urgent conditions were authored by the doctors of medicine: Mrs. Klaudia Kułak, MD and Mr. Grzegorz Honisz, MD, PhD. The Sections authored by them are: Clinical Situation (KK, GH) — Sec. 6.1.1; part of the Example (GH) — Sec. 6.2.4 and part of the Example of assigning aspirations/reservations (GH) — Sec. 6.4.1 and assigning of aspirations/reservations for the patients under consideration in the Case Study (GH). They also participated in the assessment of the results obtained (KK, GH). We are greatly thankful for their valuable professional contributions to the original paper [63].

This Chapter is structured as follows: first, we give an introduction to the decision situation, with its medical background. Then, we analyse the literature. After this, our proposed approach is described, specific optimisations problems are built, and a possible embedding framework into the existing decision making process is proposed. Then, we give some examples of how the proposed approach could be applied to the dispatching of EMS/ED services, which is then followed by a Case Study over three scenarios. We finish the Chapter by concluding and discussing the results.

As in the previous Case Study, the indexing of variables and parameters is specific only to this Chapter. This is due to the complexity of the models presented.

6.1 Introduction

Dispatching of ambulances to patients and then patients to appropriate Emergency Departments is a multi-stage decision process. Functional requirements of appropriate Decision Support System must take into account various specific medical and organisational characteristics. On one hand, in an emergency situation, help must come as soon as possible. On the other hand, however, the service must be well-suited to the patient's condition. Currently, many operational EMS dispatching strategies are used: dispatching of the closest idle unit, maximisation of the overall coverage, or maximisation of the preparedness of the EMS system [48, 14, 191]. The strategy of dispatching the closest ambulance has been proven to be sub-optimal already by Carter et al. [192] and confirmed further by other research works [193, 194].

What is more, it should be noted that in many EMS systems, ambulances differ as per the levels of speciality they can offer to patients. One example is the Polish national emergency medical system [195], where ambulances are differentiated basing on the speciality they provide to the patients. Namely, the following types of EMS units exist:

- *Basic* — ambulance with at least 2 members of staff being paramedics or nurses;
- *Specialist* — ambulance with at least 3 members of staff, one of them being a *system* doctor;
- *HEMS* — helicopter emergency medical service, with at least 3 members of staff, one of them being a *system* doctor;
- *Collaborating units* — organisations which normally do not provide public EMS services, yet might be dispatched if required (e.g., Order of Malta Ambulance Corps Poland).

In many European countries, the process of handling a medical emergency call is as follows: first, a caller dials an emergency number. All over the European Union, they can dial 112 — European general emergency number. If 112 is reached, the call would usually be taken by a non-medical dispatcher, who serves as the first triager. When the non-medical dispatcher decides that the call is medically valid, they would transfer the call to a dedicated professional medical dispatcher. The medical dispatcher would then investigate the call further, triage it appropriately and take care of assigning an appropriate EMS unit if deemed necessary. This medical dispatcher would then also help the ambulance crew to find an appropriate destination hospital. For example, such a model is present in Austria and Germany. In this model, other services (e.g., Fire Brigade) have their own dispatchers, who would be handling the call requiring their support. However, the decision problems faced by those dispatchers are out of the scope of this Chapter.

In some countries, it is also possible to bypass the 112 number and contact the professional EMS medical dispatcher directly via a dedicated number. Examples of those countries are: Poland, Romania and France. There is also another operational model (much less common than the two previous), where the call is completely handled by the non-medical 112 dispatcher. Such a model is present in Finland [196].

When present in the process, the medical dispatcher must face a decision-making problem by making a trade-off between the time requirement for the ambulance to arrive and the speciality the crew can offer to the patients. Often, this process can be facilitated by the use of dedicated Medical Priority Dispatch Software, which is discussed further in the Chapter. The software, however, helps in triaging and categorising the calls but does not optimise directly for which exact unit (in terms of its callsign) is best to respond. Decisions made may impact further treatment possibilities. For instance, dispatching an ambulance with no possibility to teletransmit the electrocardiogram (ECG) to a regional specialist centre for consultations may result in misdiagnosis of serious cardiac pathologies, including ST-Elevation Myocardial Infarction (STEMI) [197]. Thus, in the optimal decisions of ambulance-to-patient dispatching, it is required to take into account both time-to-arrival and the speciality of units.

Once the EMS unit is at the site, the team deepens the diagnosis of the patient's condition. Then, based on the results, further decisions must be made to select the appropriate Emergency Department (ED) by taking into consideration both its speciality required for the patient and the estimated time-to-arrival. In Poland, Emergency Departments are part of the national medical emergency system [195]. Just like the ambulances, EDs also offer different levels of specialities — local EDs, regional specialist centres and trauma centres. In this work, we will be referring to the two last types as *referential EDs* and the local one as *non-referential ED*. Similarly to assigning ambulances to patients, the problem of identifying the correct ED for a given patient is a nontrivial decision making process that requires establishing a trade-off between the proximity to the ED and the speciality needed in the patient's condition. According to Polish regulations, the establishment of the ED to which the patient is to be taken results from the joint collaboration of the dispatcher with the chief of the emergency medical team caring for the patient.

Some acute conditions require highly specialised quick treatment in a referential unit within a given time from symptom onset. Some examples of those are: aortic dissection (to be treated as soon as possible), STEMI (most effective treatment within 90 min. of first medical contact) or massive pulmonary embolism (most effective treatment within 48 hours of onset) [49, 50, 51]. For treatment to be effective, the patient must be transferred to the referential hospital — either directly from the scene or via re-transferring from a non-referential unit. Yet, re-transferring may add some important delays in the time-to-treatment, making further treatment difficult to be effective. Therefore, it is necessary to find an optimal patient-to-hospital assignment strategy taking into both speciality and time-to-treatment.

In this Chapter we apply our proposed generic multi-criteria Services of General Interest dispatching optimisation problem with quality-based criteria to both ambulance-to-patient and patient-to-hospital assignment problems, taking into consideration criteria such as time (cost criteria) and speciality (quality criteria) of the offered emergency service. Time and speciality requirements are not uniform across acute-state patients and depend greatly on their medical condition. We take this fact into consideration in our optimisation problems by optimising for both the time and speciality requirements of each patient individually (on a per-patient basis). In that sense, we aim to design an ambulance-to-patient and patient-to-hospital optimal assignment tool that would pinpoint the best currently possible dispatch decisions taking into consideration the clinical conditions of the patients.

Dispatch in the Chapter is understood as establishing the best assignment of precise ambulances to precise patients and further precise EDs to these patients. It is done taking into consideration the current operational state of the EMS/ED system (e.g., number of ambulances available, number of hospital beds available, time-to-arrival of a given ambulance to the patient or time to arrive at a destination hospital). This is in contrast to understanding dispatch as triage and categorisation of emergency calls, which is sometimes found in the literature. The problems proposed in this work aim to help the medical dispatcher in assigning ambulances to acute-condition patients and then the patients to emergency departments that can efficiently treat patients' conditions. The proposed problem also allows for the re-referral of patients between a non-referential and referential hospital. What is more, in this Chapter, we also propose an embedding framework of the problems proposed into the current dispatching decision-making process.

The goal of this Chapter is to analyse the importance of considering not only a single criterion (time) in the optimisation of ambulance-to-patient and patient-to-hospital assignments but also the criteria related to the speciality a given unit offers in treating a given urgent medical condition. The Chapter outlines that it is both technically possible and medically desirable to incorporate the speciality criteria in the optimisation of assignment of resources. The outcomes of our work can be used in combination with currently used call categorisation decision support software (e.g., Medical Priority Dispatch System — MPDS) and with currently existing patient transport protocols. These can be used as input to the optimisation problems proposed, enhancing the ability to assign the appropriate unit (considering both time and speciality).

To achieve this goal, we propose two multi-criteria mixed integer linear programming (MIP) optimisation problems for optimising EMS/ED assignment decisions. The first problem proposed yields a Pareto-optimal ambulance-to-patient dispatch, basing on patients' requirements on ambulances' time-to-arrival and offered speciality. These requirements are established on a per-patient basis with respect to their clinical condition. The second problem proposed yields a Pareto-optimal patient-to-hospital assignment, which also takes into consideration all patients' requirements on time-to-arrival and ED's speciality, estimated based on their clinical condition.

6.1.1 Clinical situation

To reduce the morbidity and mortality that can result from the acute phase of an illness or injury, it is essential that the ambulance response procedure is quickly ensured and that the patient is transported to the correct hospital, depending on the patient's needs and the current capacity of the emergency medical services. To do this, the patient's health condition and the maximum possible waiting time required to provide qualified medical first aid must be estimated [198]. The World's leading causes of death include cardiovascular diseases, with more than four million Europeans dying each year for that reason. According to the research conducted in 2016 — 2017 in Katowice, the most common causes for Emergency Medical Service interventions were non-traumatic internal emergencies, which most often included: hypertension, atrial fibrillation, myocardial infarction, pulmonary oedema, atrioventricular blocks, strokes, chronic obstructive pulmonary disease (COPD) and a diagnosis of bronchial asthma [199]. In addition, the most common medical emergencies include sudden cardiac arrest, which can be caused by hypoxia, cardiac tamponade, poisoning, ionic disturbances and shock. Symptoms such as abdominal pain, arm pain radiating to the jaw, unusual headache, severe bleeding, and confusion remain worrisome [200].

As mentioned, cardiac arrhythmias and cardiovascular diseases are the most common reasons for Emergency Medical Service interventions. Direct threats to life include acute coronary syndromes, pulmonary embolism or abdominal aortic aneurysms, which, if untreated, can lead to death in a short period of time. Cardiovascular diseases continue to be the world's leading causes of death, of which 50% are caused by ischaemic heart disease [201]. According to the Institute for Health Metrics and Evaluations data from 2017, 1.6 million people in Poland developed ischaemic heart disease. On the other hand, the data made available by the National Health Fund show that more than 85,000 acute coronary syndromes were recorded in Poland in 2021. Cases of acute coronary syndromes have also been reported, with nearly 67,000 myocardial infarctions [202].

Acute coronary syndromes (ACS) are mainly caused by an imbalance between the myocardial oxygen demand and its supply. The cause of the oxygen limitation is most often the presence of atherosclerotic plaque in the coronary arteries, but there may also be the presence of cardiac arrhythmias, and complications after hemorrhagic shock. ACS include ST-elevation myocardial infarction (STEMI), non-ST-elevation myocardial infarction (NSTEMI) and unstable angina

[203]. The main symptom with which patients visit the ED is sudden pain or chest tightness, usually localised retrosternally with radiation to the shoulders, angle of the jaw and elbows [204]. The diagnosis is based on the record of the received electrocardiogram, which should be performed within 10 minutes of the first contact with the health care system and on the basis of clinical symptoms. Currently, ambulances are equipped with an ECG recording machine, which allows for a quick diagnosis. If there is an ST-segment elevation, we diagnose STEMI; if there is a non-ST segment elevation, we should measure the level of troponins, elevated levels of which may indicate myocardial infarction. Once ST-segment elevation is recognised, the patient requires rapid reperfusion therapy according to the latest European guidelines or percutaneous coronary intervention (PCI) [138]. Patients diagnosed with myocardial infarction should be transported by the Emergency Medical Service to a PCI-capable facility as soon as possible. Current recommendations say that the patient should be transported to the nearest haemodynamics centre on 24-hour duty, and not to the nearest hospital. When a patient with ST-segment elevation MI (STEMI) arrives at a non-ICU hospital, he or she should be immediately transported to an invasive cardiology unit [205]. A patient presenting to a hospital where PCI can be performed should receive treatment within no more than 60 — 90 minutes if fibrinolytic treatment fails, however, the maximum delay from STEMI diagnosis to reperfusion during PCI, according to the Polish cardiac society is 120 minutes if a primary PCI strategy is chosen instead of fibrinolytic treatment. When immediate PCI is not possible, pharmacotherapy with invasive treatment should be considered, where coronary angiography is performed within 24 hours [138].

Apart from the above, nearly 5% of patients arriving in the ED are those with neurological symptoms. Sang-Beom et al. in their research distinguished a significant predominance of patients with stroke symptoms, epileptic seizures and status epilepticus among neurological emergencies. Among strokes, 80-90% of cases are patients with ischaemic stroke due to embolism or extra-cerebral vascular pathology, and underdiagnosis has been associated with increased mortality rates [206]. Ischaemic stroke is the second most common cause of death and long-term disability of adults worldwide, and the incidence of that disease increases with age. Fibrinolytic therapy is an effective treatment for stroke patients, and the therapeutic window for intravenous tissue-type plasminogen activator therapy is 3 — 4.5 hours from the onset of the first symptoms. However, only about 25% of patients on the ward receive thrombolytic treatment within the indicated time window [207, 208]. Diagnosis and implementation of treatment of patients with symptoms of acute central nervous system injury determine the effectiveness of planned therapy, but often patients arrive at an intermediate hospital that does not have a stroke unit or lacks diagnostic and therapeutic capabilities, which delays the timing of thrombolysis. In order not to delay the therapeutic window, the emergency team should notify the stroke unit staff to reduce the occurrence of in-hospital delays, while inexperienced and unequipped centres for neuroimaging in the treatment of stroke patients have an indication to use remote consultation with reference centres [209]. As a result, only appropriate hospitals can provide treatment for stroke patients.

The introduction of intravenous thrombolysis with recombinant tissue-type plasminogen activator (rtPA, alteplase) to treat acute ischaemic stroke required a revolution in the organisation of stroke care. Recognition that ”time is brain” drove effective public and prehospital awareness campaigns, such as the ”Face, Arm, Speech, Time” (FAST) test [210] and rapid prehospital triage to designated centres.

The organisation of stroke care depends upon local geography, but the implementation of dedicated acute stroke pathways varies widely. Comprehensive stroke centres provide all aspects of acute stroke care. Triage of patients eligible for endovascular thrombectomy directly to a comprehensive stroke centre (the ”mothership” model) may improve the likelihood of good outcomes, even if other hospitals are closer. Primary stroke centres are usually smaller centres that initiate intravenous thrombolysis and transfer patients eligible for endovascular thrombectomy to a comprehensive stroke centre, the so-called ”drip-and-ship” model [211]. The key aspect of any stroke service model is that patients can access specialist expertise, neuroimaging and stroke unit care without delay [212].

Worldwide, there exist accepted guidelines and dedicated protocols for the treatment of patients in life-threatening conditions and their transfer to dedicated centres. The European Resuscitation Council’s 2021 guidelines indicate that a patient suffering from a cardiac arrest should be transported to a dedicated centre for treatment of reversible causes of cardiac arrest, basing on local guidelines [213]. Local guidelines are then developed for many locations. For example, in the US, state-wide local transport protocols have been developed. These are present, for instance, in: Alabama [214] and Delaware [215]. They are briefly discussed in this Section.

In Delaware, guidelines for patients diagnosed with ST-segment elevation myocardial infarction are based on the same indications to transport the patient as soon as possible to a facility capable of performing percutaneous coronary intervention PCI with concomitant pharmacological treatment. For paediatric patients, the guidelines point to the notion of effective chest compression followed by transporting the paediatric patient from the scene to an ECMO-equipped facility as quickly as possible. Similarly, The state of Alabama has also adopted a protocol for bypassing primary care hospitals for patients with acute coronary syndromes and myocardial infarction with STEMI to hospitals with an accessible catheterisation (PCI) laboratory.

Let us now consider guidelines for stroke patients. Delaware recommends to immediately transfer a stroke patient to the nearest specialised stroke centre certified by the state of Delaware. To this end, criteria were adopted for VAN (Vision, aphasia, neglect) negative and LKW (last known well) patients with a time when they were last seen without stroke symptoms of less than 4.5 h, admission to the nearest specialised stroke centre should be considered. For VAN-positive and LKW patients of more than 4.5 h, transport of the patient directly to a certified thrombectomy centre should be considered. Similarly, the same procedures are adopted for stroke patients in Alabama.

Apart from cardiac and stroke cases, guidelines on bypassing the local facility also exist for trauma and burn patients. Patients assessed with the Glasgow Coma Score < 13 and low systolic pressure and respiratory count < 13 should be transported first to a highly specialised centre. It is also advised in Delaware that in case of an obvious injury, the patient is transported to the highest-level trauma centre. A detailed list of obvious injuries can be found in [215]. Similar guidelines on trauma handling are also found in the Alabama protocol. However, the protocol requires that the patient is diverted to the closest ED in case of: loss of airway, haemodynamic instability with no vascular access and external uncontrolled bleeding.

When it comes to burns, patients are required to be transported bypassing the nearest centre to this specialised one based on the percentage of the burn area and respiratory burns to the burn centre. Assessment of whether a given patient is to be transported to a burn centre can be made using the *rule of nines*, also given in the protocols.

There are many emergency conditions that can lead to death. Hence, it is crucial to take action in the pre-hospital setting when transporting the patient to the hospital. Many acute conditions have a therapeutic window, i.e., a maximum time to implement therapy from the time of the first worrying symptoms. Delaying appropriate medical care in a specialised unit in a serious condition practically does not guarantee survival. If a patient is transported to a hospital that has no specialised equipment and personnel, we delay the time to provide treatment at the cost of transporting the patient to a specialised centre.

6.1.2 Literature review

Organising, operating and forecasting of Emergency Medical Services is a topic of extensive research. Computer-based systems might help in making well-suited, timely decisions to support operations of the whole EMS system, e.g., in assigning ambulances to calls, assigning ambulances to EDs, ambulance routing, medical documentation handling or patient drop-off procedures and in notifications of staff required to handle a given emergency [216, 217].

Within that field, a significant number of research works focusing on the use of operational research (OR) methods for this purpose have been published. Authors of [218] identified that researchers focus on applying OR in the following problems of EMS organisation: location of ambulances with their further relocation, dispatching and routing of ambulances, the interplay of EMS with the general health system as well as forecasting of calls and availability and crew scheduling. They also note that an important research area is the development of simulation/validation tools. These observations have been backed up by the authors of another review paper [14], who also underlined the necessity of staff hiring and fleet operations optimisation and of

[48], who reviewed the problems in EMS logistics.

Some interesting usages of OR models for emergency medical system planning are given in [219, 220, 221, 222], some also investigating fairness measures [223]. A number of significant papers have also been published in the field of forecasting [224] and in the management of patients once in ED or hospital [225, 226, 227]. Those however are not directly linked to the scope of this Chapter and thus are given only as a reference for an interested reader.

From a problem statement perspective, this Chapter builds on ambulance dispatch, allocation and routing problems. These problems have been of significant research interest. Jangtenberg, with co-authors, studied the dispatching of ambulances as applied to the Dutch practice [194, 228]. Not only did they propose a new dispatch strategy outperforming the *closest idle*, but they further proposed a benchmark model for offline optimal dispatching of ambulances. EMS dispatching, taking into consideration equity call prioritisation, was studied in [229, 113], where Enayati et al. focused also on the simultaneous optimal location of ambulances. The notion of simultaneous optimisation of dispatch and location of ambulances was also applied in [10]. Authors depicted in the example of EMS data from Portugal that using OR tools with more advanced dispatch strategies can give better results than doing this by hand under *closest idle* criterion. Relocation optimisation and dispatch policies were also studied by Siong Lim et al. [230], who reviewed dynamic ambulance relocation models from the perspective of dispatch policies. Their paper also presents a comparison of different EMS dispatch policies. Boutilier et al. [231] however proposed to combine the optimisation of location and routing of ambulances in the city of Dhaka, Bangladesh.

An interesting notion in dispatch optimisation is the integration of considering different types (specialities) of ambulances [232, 233, 234], i.e., *(ALS)* — Advance Life Support and *(BLS)* — Basic Life Support which are assigned to emergency calls basing on case severity. Knight et al. [235] assess the severity with means of survival probability functions and operate the EMS system in order to maximise their expected value. As shown by Stout et al. [236], the fact of operating an all-ALS EMS system, it is possible to reduce the complexity of triaging of calls and of defining what sort of unit should respond. What is more, in such systems, there is no possible need for secondary triage on-scene (eg. calling a different ambulance type for support). This, however comes at the cost of possible prolongation of time-to-arrival and of dilution of certain paramedic skills. The latter is specifically important, since, according to Stout et al., in only 10% of calls, ALS skills are required.

To facilitate triage and categorisations of patients clinical decision support systems (CDSS) may be used. Use of such a system in the triage of patients at a Canadian paediatric Emergency Department was analysed by Michałowski et al. [237]. Specifically, when describing the current state of the art in ambulance dispatching, we should mention the general Emergency Medical Dispatch software, and specifically the Medical Priority Dispatch System (MPDS). It is a software system which aims to categorise emergency medical calls into numerical complaint-based categories and to assign them a given handling priority. The system provides the dispatcher with detailed questions, which are then asked to the caller. Basing on their answers, the system categorises the call and assigns the handling priority. Then, the calls can have a sub-group and a modifier assigned to help responders in knowing the details of the case they are to deal with. The category, priority, sub-group and modifier together form the so-called MPDS determinant [238]. The MPDS is widely used across the world and in Europe itself for triage and categorisation of the calls [239]. It has been proven that the use of MPDS system has high sensitivity but moderate to low specificity in sending appropriate units to patients requiring ALS [240, 241]. Despite this problem, Dong et al. showed that the use of an *optimised* version of MPDS in China led to an increased diagnosis consistency of the Acute Coronary Syndrome and reduced the call-to-patient arrival time [242].

The classical version of the tool, however, stops at categorising the calls and not naming (in terms of exact callsign) the best unit to respond [243]. Since optimisation methods look at identifying the best possible decisions, combining them with MPDS may be a good idea. One could first categorise the call using MPDS and then find the best exact ambulance which should respond to the call via mathematical optimisation. A similar approach was proposed in [113], where authors perform multi-criteria ambulance assignment (dispatch) optimisation considering different levels of priority of the emergency calls received. Although they do not state that the priorities are assigned using MPDS, one can easily deduce that MPDS could be a good candidate to perform this task.

After the EMS crew finishes stabilising the condition of the patient, the correct emergency department is to be identified. These problems have been studied in literature as well, mostly as ambulance routing or allocation problems. Talarico et al. [244] investigated the routing of ambulances transporting patients with different levels of acuity, yet they have not distinguished EDs basing on the speciality they can offer to patients. This has been included as an additional criterion through weighted sum scalarisation in [245]. ED competence in ambulance allocation optimisation considering possible ED overcrowding was also included by Acuna et al. [246]. The authors have considered the speciality through constraints in the optimisation problem. An important contribution in the field of emergent cases assignment to EDs was given by Leo et al. [247], where the authors included both speciality of units (as an additional criterion, with weighted sum scalarisation) combined with the ED workload management.

From the medical point of view, many patient transport protocols have been developed. These documents give guidelines to the responding teams on where to transport a given patient. Some examples of those are given for Alabama [214] and for Delaware, [215]. They give information on where and how to transport a given patient basing on certain clinical criteria. For example, in Alabama it is recommended that the ambulance crew *considers* transporting a patient with STEMI to a hospital with a catheterisation lab available. Yet, if the ambulance crew is unsure of the appropriate destination hospital, Online Medical Directors (OLMD) should be contacted for support. Similarly, in Delaware, such a patient should be transported *when practical* to a PCI-capable facility, bypassing the closest hospital. A little bit more strict are the Polish Emergency Medical System plans established for each of the 16 Polish voivodeships. As an example — in the Świętokrzyskie voivodeship, the exact addresses of hospitals capable of performing a given emergency medical procedure are named. The plan leaves choosing the most appropriate unit for a given patient X to the joint discretion of the medical dispatcher and of the chief of the medical team [248].

Unfortunately, not for all conditions such protocols exist, and not everywhere they were established. The authors of [249] outlined that 78% of US states had implemented EMS triage and destination plans for trauma, around 33% for burns, stroke, and STEMI, while only 10% for cardiac arrest. This is in line with further findings of Authors of [250], who identified only 16 states with specific transport protocols for patients with stroke caused by large vessel occlusion (LVO). What is more, even if protocols are well adopted with dedicated nationwide patient-care networks established, misdirection of patients can also happen. This is reported for European countries when referring to STEMI patients, for whom quick intervention in a PCI-capable hospital is crucial to reduce mortality [251, 252]. What is more, the protocols themselves provide guidelines on when to bypass the closest ED and transport the patient directly to a referential unit. They do not assign a given ED (in terms of its exact address) to a given, precise patient X. Neither do they take into consideration the current operational state of the EMS system, e.g., in terms of the current availability of hospital beds. That is why these protocols should be considered as input to optimisation procedures, which take care of assigning a very precise hospital to a given patient in urgency.

Our literature review outlined that there exist some currently used interesting dispatch systems (MPDS) and EMS transport protocols. Dispatch systems, however, focus mostly on performing the triage of calls and assigning a given priority to them. They do not perform the dispatch as understood by the OR community, i.e., do not give exact information on which unit (identified through its callsign) is best to respond to a given emergency. When it comes to EMS transport protocols, they give guidelines on with what sort of emergency should the ambulance crew consider taking the patient to a specialised ED. The protocols do not tell exactly that a given patient X is to be taken to hospital Y, considering the current operational state of the complete EMS system. These systems and protocols can integrate well with optimisation techniques. They can act as input guidelines — by either estimating the priority of the call or by setting standards on what speciality should the destination hospital offer to a patient suffering from a specific medical condition. Then, taking this medical input, operational research (OR) techniques can be applied to determine and assign the currently best unit to respond to an emergency (either an ambulance or an ED). Our Chapter intends to fill this gap by combining OR methods which allow for assigning exact units to exact patients in a Pareto-optimal way. This is done considering the clinical condition of the patients and the current operational state of the EMS system.

Despite the fact that OR in EMS organisation is a topic of extensive research, the majority of papers mostly consider the time criterion in the ambulance-to-patient and patient-to-hospital dispatch. There exist, however, some notable research works that also include the speciality levels of ambulances or EDs. From what we have found, it is mostly included in the optimisation problems as constraints or criterion with weighted sum scalarisation. We believe that the inclusion of speciality in the form of constraints might greatly restrict the feasible set of the problem and, in some situations, even make the dispatch infeasible. When it comes to weighted sum scalarisation, however, we believe that the assignment of appropriate weights to criteria might be a nontrivial task, especially for a medical dispatcher who is not an expert in OR. Thus, this scalarisation might not be the easiest to be applied. What is more, to the best of our knowledge, we have not identified any paper that considered possible re-referrals of patients between a unit with a lower speciality and the one with its higher level. In that sense, our Chapter intends to fill the gap identified, as well as it applies the Reference Point Method scalarisation, which we believe is well-suited for applications in Services of General Interest.

6.2 Proposed approach

Assessing acute-condition patient's emergency treatment needs vary depending on the stage of emergency medical services dispatching stage. First, the patient's condition is assessed by the medical dispatcher basing on the symptoms observed by the caller and further medical interview performed by the dispatcher. This activity is usually facilitated by using the MPDS, which provides the dispatcher with a structured questionnaire which depends on the complaint given by the caller. Based on the information collected, the dispatcher sends an ambulance basing on the estimate of the patient's condition. Then, Emergency Medical Service (EMS) crew is dispatched to the site. Once arrived, the medics deepen the diagnostics and are able to professionally assess the patient's condition. Therefore, the dispatcher's understanding of the patient's condition varies depending on the stage of the dispatching process.

Having the changing nature of information on patient's condition in mind, we propose to divide the overall dispatching problem of allocating both EMS units to patients and patients to Emergency Departments (EDs) into two distinct multi-criteria optimisation problems, i.e.,

- *EMS Dispatching Problem* (P1) — problem of assigning adequate ambulances (EMS) to patients by taking into account initial patients' conditions given by the caller;

- *ED Dispatching Problem* (P2) — problem of assigning patients to adequate emergency departments by taking into account more actual patients' conditions assessed on site by the EMS crew.

In this work, we focus on acute cardiac conditions. Thus, in the remainder of the Chapter, whenever we refer to the *level speciality*, it means specifically cardiological speciality. Yet, the approach and following formulations are general enough to be used directly whenever referring to any other possible medical emergency and speciality in treating any other condition.

In the proposed approach, we specifically consider the speciality of both EMSes and EDs. For modelling purposes, let us assume that the level of speciality is given by a real number

$$s \in [0; 1], \tag{6.1}$$

where $s = 0$ means no cardiological speciality offered at all, and $s = 1$ means the best cardiological unit in the area. While taking the example of EMS dispatching, those two extreme values could mean, for example, a taxi ($s = 0$) and a mobile intensive care unit ($s = 1$) [253]. Similarly, in the ED dispatching problem $s = 0$ could mean a general practice nurse office and $s = 1$ super specialised cardiology hospital. It is worth mentioning that we deliberately decide to model the level of speciality as a real number in the interval given in (6.1), rather than by a set of discrete choices as in Polish legislation (e.g., ambulances *P, S, HEMS*). This is to better model varieties in speciality, taking into consideration for instance, different equipment onboard the ambulances or differences in the crew's experience in treating cardiac conditions.

By representing the speciality as a vector, one could also consider specialities towards treating multiple medical conditions simultaneously, e.g., $\boldsymbol{s} = [s_1, s_2, \ldots, s_n]$, where s_1 — level of cardiological speciality, s_2 — level of neurological speciality, s_3 — level of orthopaedic speciality and so on. Yet, such a setup is out of the scope of this Chapter.

6.2.1 EMS Dispatching Problem (P1)

In this Section we propose a multi-criteria mixed-integer linear programme (MIP) optimisation model for assigning ambulances to patients, taking into account both speciality of the unit dispatched and time-to-arrival. The model of the EMS Dispatching Problem is given in (6.2).

$$
\begin{aligned}
\max \quad & [s_1^1, -t_1^1, \dots, s_{\overline{p}}^1, -t_{\overline{p}}^1] \quad \overline{p} = |\mathcal{P}| \\
\text{s.t.} \quad & s_p^1 = \sum_{a \in \mathcal{A}} s_a \, y_p^a \qquad \forall p \in \mathcal{P}, \\
& t_p^1 = \sum_{a \in \mathcal{A}} t_p^a \, y_p^a \qquad \forall p \in \mathcal{P}, \\
& \sum_{a \in \mathcal{A}} y_p^a = 1 \qquad \forall p \in \mathcal{P}, \\
& y_p^a \in \{0; 1\} \qquad \forall p \in \mathcal{P}, \forall a \in \mathcal{A}
\end{aligned}
\tag{6.2}
$$

where:

- s_p^1 — speciality received by patient p through EMS dispatch (variable);
- t_p^1 — time it takes for dispatched ambulance to reach patient p (variable);
- s_a — speciality level of ambulance a, $s_a \in [0, 1]$ (parameter);
- t_p^a — time needed to reach patient p by ambulance a;
- $\mathcal{A}$ — set of available ambulances (parameter);
- $\mathcal{P}$ — set of patients needing support;
- y_p^a — binary variable describing assignment of ambulance a to patient p; takes 1 when a is assigned to p and 0 otherwise.

The problem by design is to make the dispatcher's decisions easier on choosing the assignment of available ambulances to patients after receiving the emergency calls. Therefore, we assume that the patients are known a priori and that the number of calls is lower than the number of available ambulances. In the case when a new emergency call appears when no ambulance is available, it should be handled later on, after some ambulances become idle. However, it is possible to extend the decision model by including queuing theory constraints, as proposed in [113]. This is deliberately omitted in this work, since our goal is to outline the importance and performance of the multi-criteria strategy, as opposed to standard strategies.

6.2.2 ED Dispatching Problem (P2)

This Section gives the multi-criteria MIP formulation of the ED Dispatching Problem in (6.3). It is to be solved in the second step, after solving EMS Dispatching Problem, once the initial assessment made through the medical interview has been adjusted or confirmed by the emergency crew at the site. ED Dispatching allows us to determine the dispatch of ambulances (with patients) to emergency departments. The problem gives the possibility to re-refer patients from a less specialised department to a more specialised one. The formulation takes into consideration the fact that admitting the patient first to a non-referential hospital and then to a referential one may boost the level of speciality received by the patient. This is due to the fact that some pre-treatment might be given to the patient in the non-referential unit. The factor by which the pre-treatment participates in total treatment is given by the arbitrary parameter η_1.

$$
\begin{aligned}
\max \quad & [s_1^2, -t_1^2, ..., s_{\overline{p}}^2, -t_{\overline{p}}^2] \quad \overline{p} = |\mathcal{P}| \\
\text{s.t.} \quad & s_p^2 = s_{p1}^2 + s_{p2}^2 + s_{p3}^2 + s_{p4}^2 \qquad \forall p \in \mathcal{P}, \\
& s_{p1}^2 = \eta_1 \, (\sum_{h1 \in \mathcal{H}_{\text{not}}} s_{h1} \, y_p^{h1}) \qquad \forall p \in \mathcal{P}, \\
& s_{p2}^2 = (1 - \eta_1) \, (\sum_{h1 \in \mathcal{H}_{\text{not}}} w_p^{h1} \, s_{h1}) \qquad \forall p \in \mathcal{P}, \\
& s_{p3}^2 = \sum_{h2 \in \mathcal{H}_{\text{ref}}} s_{h2} \, b_p^{h2} \qquad \forall p \in \mathcal{P}, \\
& s_{p4}^2 = \sum_{h2 \in \mathcal{H}_{\text{ref}}} s_{h2} \, u_{p,h1}^{h2} \qquad \forall p \in \mathcal{P}, \\
& t_p^2 = t_{p1}^2 + t_{p2}^2 + t_{p3}^2 \qquad \forall p \in \mathcal{P}, \\
& t_{p1}^2 = \sum_{h1 \in \mathcal{H}_{\text{not}}} t_p^{h1} \, y_p^{h1} \qquad \forall p \in \mathcal{P}, \\
& t_{p2}^2 = \sum_{h2 \in \mathcal{H}_{\text{ref}}} t_p^{h2} \, b_p^{h2} \qquad \forall p \in \mathcal{P}, \\
& t_{p3}^2 = \sum_{h1 \in \mathcal{H}_{\text{not}}} \sum_{h2 \in \mathcal{H}_{\text{ref}}} g_{h1,p}^{h2} \, u_{h1,p}^{h2} \qquad \forall p \in \mathcal{P}, \\
& w_p^{h1} \leq y_p^{h1} \qquad \forall p \in \mathcal{P}, \ \forall h1 \in \mathcal{H}_{\text{not}}, \\
& w_p^{h1} \leq 1 - \sum_{h2 \in \mathcal{H}_{\text{ref}}} u_{p,h1}^{h2} \qquad \forall p \in \mathcal{P}, \ \forall h1 \in \mathcal{H}_{\text{not}}, \\
& w_p^{h1} \geq y_p^{h1} + (1 - \sum_{h2 \in \mathcal{H}_{\text{ref}}} u_{p,h1}^{h2}) - 1 \quad \forall p \in \mathcal{P}, \ \forall h1 \in \mathcal{H}_{\text{not}}, \\
& \sum_{p \in \mathcal{P}} y_p^{h1} \leq \overline{H_{h1}} \qquad \forall h1 \in \mathcal{H}_{\text{not}}, \\
& \sum_{p \in \mathcal{P}} b_p^{h2} + \sum_{p \in \mathcal{P}} \sum_{h1 \in \mathcal{H}_{\text{not}}} u_{h1,p}^{h2} \leq \overline{H_{h2}} \qquad \forall h2 \in \mathcal{H}_{\text{ref}}, \\
& u_{h1,p}^{h2} - y_p^{h1} \leq 0 \qquad \forall p \in \mathcal{P}, \forall h1 \in \mathcal{H}_{not}, \forall h2 \in \mathcal{H}_{ref}, \\
& \sum_{h_2 \in \mathcal{H}_{ref}} u_{h1,p}^{h2} \leq 1 \qquad \forall p \in \mathcal{P}, \forall h1 \in \mathcal{H}_{not}, \\
& \sum_{h_1 \in \mathcal{H}_{not}} y_p^{h1} + \sum_{h_2 \in \mathcal{H}_{ref}} b_p^{h2} = 1 \qquad \forall p \in \mathcal{P}, \\
& y_p^{h1} \in \{0; 1\} \qquad \forall p \in \mathcal{P}, \forall h1 \in \mathcal{H}_{\text{not}},
\end{aligned}
\tag{6.3}
$$

$$b_p^{h2} \in \{0;1\} \qquad \forall p \in \mathcal{P}, \forall h2 \in \mathcal{H}_{\text{ref}},$$
$$u_{h1,p}^{h2} \in \{0;1\} \qquad \forall p \in \mathcal{P}, \forall h1 \in \mathcal{H}_{\text{not}}, \forall h2 \in \mathcal{H}_{\text{ref}}$$

where:

- s_p^2 — speciality received by patient p through ED dispatch. Is the sum of four components, namely $s_{p1}^2, s_{p2}^2, s_{p3}^2, s_{p4}^2$ (variables);
- t_p^2 — time it takes for patient p to reach final ED destination. Is the sum of three components, namely $t_{p1}^2, t_{p2}^2, t_{p3}^2$ (variable);
- $s_{h1/h2}$ — speciality offered by ED $h1 \in \mathcal{H}_{not}$ or by $h2 \in \mathcal{H}_{ref}$ (parameter);
- $\mathcal{H}_{ref}$, $\mathcal{H}_{not}$ — sets of available emergency departments - referential and non-referential respectively;
- η_1 — factor by which patient is treated by the first emergency department, $\eta_1 \in [0;1]$ (parameter);
- $t_p^{h1/h2}$ — time needed to drive patient p to ED $h1 \in \mathcal{H}_{not}$ or to $h2 \in \mathcal{H}_{ref}$ (parameter);
- $g_{h1,p}^{h2}$ — time needed to re-refer patient p from non-referential ED $h1 \in \mathcal{H}_{not}$ to referential ED $h2 \in \mathcal{H}_{ref}$ (parameter);
- $\overline{H_{h1/h2}}$ — maximum available capacity of emergency ED $h1 \in \mathcal{H}_{not}$ or by $h2 \in \mathcal{H}_{ref}$ at dispatch time (parameter);
- y_p^{h1} — binary variable describing assignment of non-referential hospital $h1 \in \mathcal{H}_{not}$ to ambulance transporting patient p;
- b_p^{h2} — binary variable describing assignment of referential hospital $h2 \in \mathcal{H}_{ref}$ to ambulance transporting patient p, direct transport to the referential hospital;
- $u_{h1,p}^{h2}$ — binary variable describing re-referral of patient p from ED $h1 \in \mathcal{H}_{not}$ to ED $h2 \in \mathcal{H}_{ref}$;
- w_p^{h1} — linearisation variable of binary product: $y_p^{h1}\,(1 - \sum_{h2\in\mathcal{H}_{\text{ref}}} u_{p,h1}^{h2})$ (variable).

Derivations of speciality and time-to-treatment delivered to the patient p are given in constraints on s_p^2 and on t_p^2. Constraints on w_p^{h1} ensure the linearisation of the binary product of $w_p^{h1} = y_p^{h1}\,(1 - \sum_{h2\in\mathcal{H}_{\text{ref}}} u_{p,h1}^{h2})$, which is derived in order to get delivered speciality equal to $s_{h_1}, h_1 \in \mathcal{H}_{not}$ if p is not re-referred to $h_2 \in \mathcal{H}_{ref}$. If the re-referral happens, the calculated speciality delivered will be equal to $\eta_1 s_{h_1} + s_{h_2},\ h_1 \in \mathcal{H}_{not}, h_2 \in \mathcal{H}_{ref}$. Constraints with $\overline{H_{h1/h2}}$ assure that the current capacities of EDs are not violated. Constraint $u_{h1,p}^{h2} - y_p^{h1} \leq 0$ assures that it is only possible to re-refer p from $h_1 \in \mathcal{H}_{not}$ to $h_2 \in \mathcal{H}_{ref}$, if p was first transported directly to h_1. Formulations $\sum_{h_2 \in \mathcal{H}_{ref}} u_{h1,p}^{h2} \leq 1$ assures that p must be taken directly to exactly one of hospitals $h_1 \in \mathcal{H}_{not}$ or $h_2 \in \mathcal{H}_{ref}$ and $\sum_{h_1 \in \mathcal{H}_{not}} y_p^{h1} + \sum_{h_2 \in \mathcal{H}_{ref}} b_p^{h2} = 1$ that p can be re-referred to maximum one $h_2 \in \mathcal{H}_{ref}$, yet this is not mandatory.

6.2.3 Embedding into current decision process

It is possible to embed the optimisation models given in Sec. 6.2.1 and in Sec. 6.2.2 into the standard decision process of the dispatcher, rather than completely re-organising it. Since both optimisation problems are multi-criteria, to solve them, DM's preferences towards all criteria should first be estimated [104]. In the proposed approach, criteria are associated with each patient's medical condition and the dispatcher takes the role of the DM. Having the above in mind, we propose that preferences are given through estimation of *reservations* and *aspirations* towards all criteria. In that way, it is possible to reflect preferences as a direct function of the patient's condition, rather than through hard to understand and to explain weights. For the reasoning behind it please consult the findings presented in Chapter 3.

The integration schematic framework is shown in Fig. 6.1. The additions proposed in this Chapter are shown as green rectangles. Standard process elements are shown as blue rectangles and orange ellipses are used for starting and ending events.

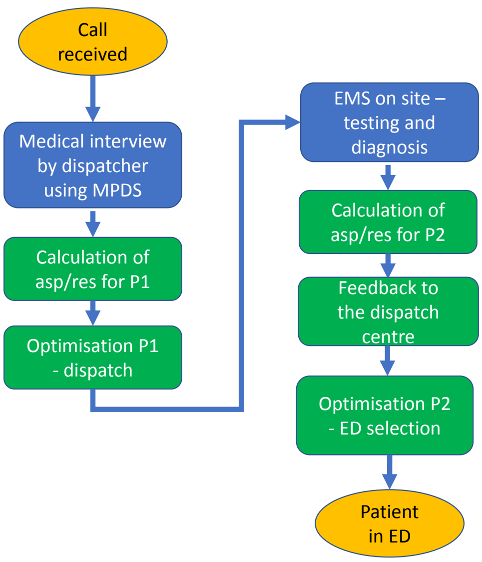

Figure 6.1: Proposed embedding framework in current decision process

The starting point of the decision making problem is the receipt of an emergency call by the dispatcher. We propose to take advantage of the medical interview supported by MPDS. After some adjustments, the MPDS (or similar system) could be leveraged to calculate aspirations/reservations for EMS Dispatching problems basing on the symptoms given by the caller. Once they are known, the optimisation takes place and a Pareto-optimal EMS crew is dispatched.

After that, we propose that after EMS' arrival and additional diagnostics, the emergency team use their portable tablets/computers, such as the ones currently being part of the State Command Support System for the State Emergency Medical System — *SWD PRM* [254]. In this process, the existing patient transport protocols can be embedded in the SWD PRM system to calculate aspirations and reservations basing diagnostics performed by the crew. Taking into consideration the aspiration and reservation values calculated, optimisation takes place in the dispatch centre in order to identify a Pareto-optimal emergency department for the patient's condition. The process stops when the patient arrives at the ED.

6.2.4 Example

Let us now demonstrate an example of how the proposed decision process can work in the practice of EMS/ED dispatch. Note that to better show the decision making process of our proposed solution in the example we calculate aspirations/reservations in a truly simplified way. In a real-life situation, the dispatcher should ask many more detailed questions, possibly following the recommendations of the MPDS. This would result in a much more granular way of calculating the values. The goal of this Section is only to give the reader the feeling of how the proposed solution could be put into action, hence the example is very simple and straightforward. Let us consider the following situation:

1. *Medical dispatcher receives an emergency call. The caller gives the following patient's symptoms in the course of the medical interview: acute chest pain radiating to the left arm, conscious, and breathing.* Since symptoms can be significant for an acute myocardial infarction, this call is treated as urgent and as requiring specialist EMS care. Therefore, the interactive questionnaire (or modified MPDS) estimates the aspiration for EMS arrival at the site ($a_{t_{EMS}}$) to 7 min. and the reservation for EMS arrival at the site ($r_{t_{EMS}}$) to 14 min. Since possible teletransmission of ECG to the specialist centre might be required, aspiration towards EMS speciality ($a_{s_{EMS}}$) is estimated to 0.9 and reservation ($r_{s_{EMS}}$) to 0.7. Therefore, for this particular patient, we require an ambulance that would arrive in a time shorter than 14 minutes and ideally in 7 mins, offering a speciality greater than 0.7 and ideally 0.9.

2. *EMS is dispatched according to the preferences described in pt. 1 and arrives within 10 mins. Once the crew arrives at the scene they confirm the symptoms given by the caller. ECG with teletransmission to the on-duty cardiologist is performed which reveals significant ST segment changes. Basing on them, a pre-hospital diagnosis of ST-elevation myocardial infarction is made.* STEMI is a condition that should be optimally treated in

a highly specialised cardiologist centre within 120 mins. Thus, the following aspirations and reservations towards time-to-arrival and speciality of the ED are calculated using the EMS crew's portable computer are: $a_{t_{ED}} = 20, r_{t_{ED}} = 120, a_{s_{ED}} = 0.9, r_{s_{ED}} = 0.8$. Thus, it is required that this patient arrives at an ED with a speciality greater than 0.8, ideally 0.9 and in a time shorter than 120 mins, ideally in 20 mins. Please note that aspiration/reservation towards the time criterion was estimated such that they include the total time-to-arrival/time-to-treatment. These values are then fed back to the dispatch centre for Pareto-optimal assignment of the Emergency Department. Once established, the EMS crew take the patient to the ED chosen.

The points above illustrate the idea of how the process could be seen within a realistic example. Once the values of aspirations and reservations are calculated, the optimisation happens taking them as the decision model. Calculation of aspiration/reservation values for other patients is analogous to the case presented. More information on how this can happen is given in

Please note also that sometimes it may be possible to assign units by giving better final values of criteria than the value of the aspiration levels for some of the patients, and not worsening the results for the others. Such a dispatch will be selected by the optimiser.

6.3 Relation with SGI dispatch optimisation problem

Both problems proposed in this Case Study are specific implementations of the SDOQ problem (3.1). In the problems proposed, we consider only patients' criteria. This is due to the fact that assigning of ambulances to patients, and then patients to EDs does not happen on any free market (as we aim to save lives). In both problems (P1 and P2) we consider two criteria per each patient — one cost ($t_p^{1/2}$) and one quality criterion ($s_p^{1/2}$). They are the specific formulations of the generic functions $f^1(\boldsymbol{x})$ and $f^2(\boldsymbol{x})$ from the SDOQ problem. The cost criterion in this work is understood as the time it takes for a patient to get appropriate medical aid, and the quality one as the speciality a given ambulance/ED offers in treating the emergency. The general vector $\boldsymbol{x}$ in Problem 1 is composed of the assignment variables y_p^a and in the Problem 2 of variables $y_p^{h1}, b_p^{h2}, u_{h1,p}^{h2}$. Since both problems are related to assignment, the demand for the service Δ_i from SDOQ equals 1. In P1 no specific constraints or variables are considered for the $\mathcal{Q}$ set, as all constraints appear in the SDOQ problem already. For the P2 however, the set $\mathcal{Q}$ is described by multiple additional constraints and the linearisation variable w_p^{h1}. Those criteria and variables model the re-referral process of the patient from a non-referential hospital to a referential one.

Contrarily to Case Study I, this time we deal with an assignment problem, and not a balancing one. This is why all the assignment variables are binary and not continuous as previously. However, similarly to the previous case we choose to scalarise the problems using the Reference Point Method scalarisation in the non-equitable form. As before, we decide to use the non-equitable form as our previous experiments in Sec. 4 outlined that the solution time of the equitable one is of orders of magnitude higher than the one of the non-equitable one and the possible loss of performance is much less significant.

Despite our decision to proceed with the non-equitable problem, for a real-life situation, a conscious DM's decision should be made on with which formulation to go. As some of the computational issues could potentially be addressed through the means of some cloud computing resources. Unfortunately, these are not available for this work.

6.4 Case Study

We test the proposed approach in simulations. To make them more viable we take the real-life activations of the American EMS, from the 2020 National Emergency Medical Services Information System (NEMSIS) Public-Release Research Data Set [255]. In this data set extensive information on the situation of the system at the moment of the call and on the health condition of the urgent patient is given. This includes both symptoms given by the caller to the dispatcher and the diagnosis made by the EMS crew once arrived at the scene.

For our analysis, we take data on 41 patients with cardiac conditions coming from the data set. For this, we take the values of the following attributes for each patient from the set of 41 considered:

- primary symptom;
- provider's primary impression;
- complaint reported by Dispatch to EMS (understood as symptoms given by the caller);
- flag if cardiac arrest happened;
- cardiac arrest aetiology;
- first monitored arrest rhythm of the patient;
- reason why the cardio-pulmonary resuscitation (CPR) was stopped;
- end of EMS cardiac arrest event;
- age;
- possible injury;
- systolic blood pressure (BP);
- Peripheral oxygen saturation (SpO2);

- respiratory rate;
- heart rate;
- End-tidal carbon dioxide (EtCO2);
- pain scale score;
- ECG type;
- level of responsiveness (AVPU scale);
- stroke scale score;
- Glasgow coma scale;
- cardiac rhythm (coming from ECG).

Detailed information on the attributes available in the NEMSIS data set can be found in the NEMSIS Data Dictionary [256].

For all 41 patients considered we arbitrarily assigned aspiration and reservation values towards both time and speciality for both problems considered (P1 and P2). This was done basing on an expert knowledge assessment of the cases by Dr. G. Honisz, basing on attribute values given in the data set.

The optimisation problems considered were coded in Matlab using CVX, a package for specifying and solving convex programs [145, 146] and solved using Gurobi.

6.4.1 Example of assigning aspirations/reservations

This Section describes an example of assigning values of aspirations and reservations for the two problems considered (P1 and P2). For this we give the rationale for assigning those values to two cases of acute condition patients — Patient A and Patient B. Values of **selected** attributes from the list are given in Tab. 6.1. Due to text length limitations, in this Chapter, we deliberately show only selected attributes in this example, since not all of them are directly relevant in assigning the values of aspirations/reservations for these particular patients. One should remember that this activity is very case-specific and depending on the condition itself different vital parameters will be taken into account. What is more, all information regarding cardiac arrest is dropped since it did not occur in the discussed patients. However, the complete data on the patients considered is available in the online repository `https://bit.ly/3rPZATG`.

Table 6.1: Example patients' attributes

Parameter	**Patient A**	**Patient B**
Primary symptom	Chest Pain, Unspecified	Chest Pain, Unspecified
Complaint Reported by Dispatch to EMS	Transfer/Interfacility/Palliative Care	Heart Problems
Age	67 years	47 years
Possible injury	No	No
Systolic BP	109	110
Heart rate	81	64
Respiratory rate	18	12
SpO2	96	100
ECG type	12 Lead-Left Sided	4 Lead
Level of Responsiveness (AVPU)	Alert	Alert
Glasgow Coma Score	15	15
Cardiac rhythm	STEMI Anterior Ischaemia	STEMI Inferior Ischaemia

The estimated aspiration and reservation values for both problems (EMS Dispatching and ED Dispatching) for patients A and B are given in Tab. 6.2.

Table 6.2: Aspiration and reservation values of the exemplary patients

	Patient A and **Patient B**
$a_{t_{EMS}}$ [min.]	7
$r_{t_{EMS}}$ [min.]	14
$a_{s_{EMS}}$	0.90
$r_{s_{EMS}}$	0.70
$a_{t_{ED}}$ [min.]	20
$r_{t_{ED}}$ [min.]	120
$a_{s_{ED}}$	0.90
$r_{s_{ED}}$	0.80

The primary symptom shown in both patients is chest pain, not related to any trauma. What is more, In Patient B the problem was assessed by the dispatcher as related to the heart. Patient A, however, was being transferred between facilities. Given the nature of the primary symptom, plus information in the complaint strict values for P1 speciality and time-to-arrival aspiration and reservation values were assigned. Since the Case Study presented is only an example of the performance of the multi-criteria method as opposed to other benchmarks with the same patients considered, we assigned the values of aspirations/reservations basing on the primary complaint. This was also done in that way due to the fact that not all important data are available in the

NEMSIS data set for the patients studied. In a real-life situation, the dispatcher should take into consideration more aspects before assigning the aspiration/reservations. This limitation does not impact the conclusions made, since we take the very same patients for all techniques considered.

As assessed by the EMS on site, both patients considered suffer from ST-segment elevation myocardial infarction (STEMI). The significance of total ischaemic time in the context of STEMI is very important. Prolonged total ischaemic time is a problem not specific to a certain geography or population, it exists across the world with varying degrees of intensity. Total ischaemic time strongly correlates as an independent predictor of major adverse cardiovascular events (MACE). Shorter ($<$ 3h) total ischaemic time is associated with a reduced risk of mortality.

One of the underlying mechanisms of increased mortality with prolongation of ischaemic time is that infarct size significantly affects myocardial tissue and keeps on damaging with every passing second of ischaemic time. Prolonged total ischaemic time associates with higher mortality of STEMI patients in whom the recommended *door-to-balloon* is achieved. Hence, even with optimal reperfusion (primary PCI), prolonged ischaemic time may cause higher mortality and less myocardial salvage. A decrease in door-to-balloon time is unlikely to render the ultimate desired reduction in mortality after primary coronary angioplasty.

As given, the treatment is based on primary PCI. This can however be only delivered by a cardiologist in a highly specialised invasive cardiology hospital unit. Taking into consideration the above reasoning, strict aspiration/reservation values for P2 speciality and time-to-treatment values were assigned.

6.4.2 Numerical results: EMS Dispatching Problem (P1)

The approach proposed in this Chapter is tested in simulations. For the test setup, we took 41 real-life acute-state cardiac patients from the NEMSIS data set. This Section presents the simulation results applied to Problem 1, i.e., EMS Dispatching Problem. For testing purposes, we assumed that 45 ambulances are available to respond to the calls since queuing models are not considered in the scope of current work. Time-to-arrival for each ambulance to each patient was chosen randomly from the uniform distribution $t_p^a \in [6; 200]$ min. and speciality of the same from uniform distribution $s_a \in [0; 1]$. Aspirations and reservations $a_{t_{EMS}}, r_{t_{EMS}}, a_{s_{EMS}}, r_{t_{EMS}}$ were assessed using expert knowledge by Dr. G. Honisz, taking into account patient's condition as described in the NEMSIS data set.

The optimisation results obtained through solving Problem 1 are compared with results of two other goal functions - minimisation of total time-to-arrival and weighted sum aggregation, where two criteria are considered, i.e., minimisation of total time-to-arrival and maximisation of total speciality delivered. Those are given in (6.4) and (6.5)

$$\begin{aligned}\min \quad & \sum_{p\in\mathcal{P}} t_p^1 \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{Q}\end{aligned} \tag{6.4}$$

$$\begin{aligned}\min \quad & v_1 \sum_{p\in\mathcal{P}} t_p^1 - v_2 \sum_{p\in\mathcal{P}} s_p^1 \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{Q}\end{aligned} \tag{6.5}$$

where: v_1 and v_2 — arbitrarily chosen weights for the weighted sum aggregation, $\mathcal{Q}$ — feasible set of the problem P1 and $\boldsymbol{x}$ —- vector of decision variables.

Results obtained by solving the EMS Dispatching Problem are shown in Tab. 6.3. Similarly to all previous experiments, we analyse the results on the following metrics:

1. Number of criteria being at least as good as their reservations associated (max. 82);
2. Number of criteria being worse from their reservations by at least 10%;
3. Maximum percentage gap between the criterion value and its reservation, for both cost and quality criteria jointly;
4. Mean value of patients' utility functions;
5. Value of s_p^1 received by Patient 9 ($a_{s_{EMS}} = 0.60, r_{s_{EMS}} = 0.40$);
6. Value of t_p^1 received by Patient 9 ($a_{t_{EMS}} = 15, r_{t_{EMS}} = 50$);
7. Value of s_p^1 received by Patient 3 ($a_{s_{EMS}} = 0.40, r_{s_{EMS}} = 0.20$);
8. Value of t_p^1 received by Patient 3 ($a_{t_{EMS}} = 30, r_{t_{EMS}} = 120$).

Table 6.3: EMS Dispatching Problem results

EMS Dispatching Problem (P1)					
Index	(6.2)	(6.4)	(6.5) $v_1 = 1$, $v_2 = 1$	(6.5) $v_1 = 0.8$, $v_2 = 15$	(6.5) $v_1 = 15$, $v_2 = 0.8$,
1	73.00	57.00	57.00	61.00	57.00
2	4.00	17.00	17.00	14.00	17.00
3	41.45%	99.65%	99.65%	99.65%	99.65%
4	−43.85	−377.24	−377.24	−269.41	−377.24
5	0.60	0.22	0.22	0.22	0.22
6	32.38	16.04	16.04	16.04	16.04
7	0.21	0.50	0.50	0.50	0.50
8	46.51	6.03	6.03	6.03	6.03

As can be seen from the results, only when the proposed approach is applied (P1: EMS Dispatching Problem) all overall indices studied (no. 1 to 4) perform better than in other approaches. Especially the average value of the utility function is importantly higher than in the other approaches.

Aspirations and reservations are assigned individually for a given clinical condition, basing on medical knowledge. These values vary between clinical conditions. Taking aspirations/reservations into consideration in ambulance dispatch optimisation allows for assigning ambulances in a way that they are met. As a result, the problem developed focuses on ambulance dispatch basing on patients' condition and not simply treating all patients alike, as is done in the alternative approaches studied. In that regard, some ambulances might be chosen that are further away from the calls (yet still within an acceptable distance), but offer better speciality. In that sense, the assignment result is more fit-for-purpose given the current operational state of the EMS system. We can thus conclude that the proposed generic multi-criteria Services of General Interest dispatching optimisation problem with quality-based criteria managed to yield results, which are more tailored to patients' needs in the presented EMS dispatching Case Study.

6.4.3 Numerical results: ED Dispatching Problem (P2)

In this Section, we present the numerical results of the proposed ED Dispatching Problem (P2) as obtained by optimising the test case. For this reason, we take the very same patient cases as in the Case Study for P1, yet this time by taking into optimisation the values of $a_{t_{ED}}, r_{t_{ED}}, a_{s_{ED}}, r_{t_{ED}}$. They were estimated using expert knowledge by Dr. G. Honisz, taking into consideration patients' condition as assessed by the EMS crew on scene and reported in the NEMSIS data set.

Similarly to the P1 Case Study, we compare the hospital dispatch obtained through solving the proposed problem with two other objective functions approaches, i.e.,

$$\begin{aligned} \min \quad & \sum_{p\in\mathcal{P}} t_p^2 \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{Q} \end{aligned} \tag{6.6}$$

$$\begin{aligned} \min \quad & v_{12}\sum_{p\in\mathcal{P}} t_p^2 - v_{22}\sum_{p\in\mathcal{P}} s_p^2 \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{Q} \end{aligned} \tag{6.7}$$

where: v_{12} and v_{22} — arbitrarily chosen weights for the weighted sum aggregation, $\mathcal{Q}$ — feasible set of the problem P2 and $\boldsymbol{x}$ —- vector of decision variable.

We look at thte same indices as in Case Study I with the addition of two-case specific ones — namely no. 9 and 10:

1. Number of criteria being at least as good as their reservations associated (max. 82),
2. Number of criteria being worse from their reservations by at least 10%;
3. Maximum percentage gap between the criterion value and its reservation, for both cost and quality criteria jointly;
4. Mean value of patients' utility functions;
5. Value of s_p^2 received by Patient 9 ($a_{s_{ED}} = 0.70, r_{s_{ED}} = 0.55$);
6. Value of t_p^2 received by Patient 9 ($a_{t_{ED}} = 50, r_{t_{ED}} = 120$);
7. Value of s_p^2 received by Patient 3 ($a_{s_{ED}} = 0.50, r_{s_{ED}} = 0.35$);
8. Value of t_p^2 received by Patient 3 ($a_{t_{ED}} = 90, r_{t_{ED}} = 120$);
9. Number of direct transports to the referential hospital;
10. Number of re-referrals to the referential hospital.

The ED Dispatching Problem proposed allows for the differentiation of referential and non-referential hospitals. By design, it helps the dispatcher to decide whether to dispatch the patient directly to a non-referential or referential unit, as well as to whether first dispatch them to a non-referential hospital and then to re-refer them to a referential unit. Of course, such a re-referral comes at an increased time for the patient to reach their final ED destination.

To test the behaviour of the ED Dispatching Problem proposed under different decision situations (operating conditions) in terms of re-referrals, we test the approach under three scenarios, namely:

- **Scenario 1 (S1)** — both re-referrals and direct transport to referential hospitals are possible under *normal operating conditions*, where it is time-consuming to re-refer patients from a non-referential to a referential hospital;
- **Scenario 2 (S2)** — direct transports of patients to referential hospitals are not possible;
- **Scenario 3 (S3)** — both re-referrals and direct transport to referential hospitals are possible, yet the time of re-referral is assumed very little.

In the test case, we assume the existence of four hospitals in the considered EMS operating region - three non-referential (H1, H2, H3) and one referential (H4). Assumed speciality values on treating cardiovascular diseases of those hospitals are given in Tab. 6.4. The current capacities of EDs are assumed to be 20 patients for non-referential hospitals and 10 patients for the referential one. Queuing in ED is not considered since it is deemed out of the scope of this work. All time values, i.e., time-to-treatment of a given patient and times of re-referrals between hospitals were chosen randomly. Due to space limitations we do not cite the test values in the Chapter, yet all data used are available in the online storage `https://bit.ly/3rPZATG`. For all testing cases we assumed $\eta_1 = 0.2$. This parameter is an arbitrary value, which says what percentage of non-referential hospital's treatment capabilities is added for a patient, who is transferred further to a referential unit. It is only taken into consideration if the optimiser decides that a re-transfer between hospitals is needed, if not then it does not impact the speciality. If the value of the parameter is close to 1, the total speciality received by a re-transferred patient will be close to the sum of specialities offered by the non-referential and the referential units. Yielding specialities generally higher than required. This however will be coming at a large cost for time for re-transferring and not meeting the time reservation. By analogy, if it is close to 0 the speciality received will be close to the speciality offered by the referential facility and direct transfers will be preferred. This is because the non-referential unit would have very little impact on the overall treatment and re-transfer would worsen the time criterion. Having said the above, there is no clear linear relation between the number of re-transferred patients and the value of η_1. As a good practice and to reflect the clinical reality of re-transfers, we propose to keep it between 0.1 and 0.3, as always some sort of treatment will be applied (better stabilisation of condition or deepening of diagnostics).

Table 6.4: Assumed speciality of hospitals

	Speciality
H1	0.18
H2	0.35
H3	0.56
H4	0.88

6.4.4 Scenario 1

In this Section, we give numerical results obtained by optimising the proposed ED Dispatching Problem under Scenario 1. These results are then compared with the optimisation of goal functions (6.6) and (6.7). Those are shown in Tab. 6.5.

Table 6.5: ED Dispatching Problem results (S1)

ED Dispatching Problem (P2) Scenario 1 (S1)					
Index	(6.3)	(6.6)	(6.7) $v_1 = 1$, $v_2 = 1$	(6.7) $v_1 = 0.8$, $v_2 = 15$	(6.7) $v_1 = 15$, $v_2 = 0.8$,
1	74.00	58.00	58.00	62.00	58.00
2	8.00	24.00	24.00	20.00	24.00
3	36.36%	77.50%	77.50%	77.50%	77.50%
4	−83.77	−470.23	−470.23	−404.35	−470.23
5	0.95	0.56	0.56	0.56	0.56
6	32.54	17.64	17.64	17.64	17.64
7	0.35	0.18	0.18	0.18	0.18
8	36.96	28.99	28.99	28.99	28.99
9	5.00	10.00	10.00	10.00	10.00
10	4.00	0.00	0.00	0.00	0.00

Similarly to the EMS Dispatching Problem (P1), the ED Dispatching Problem gave results, which outperformed the other approaches considered, in the overall indices (no. 1 to 4). Especially improvement was seen for the mean value of the utility function (index no. 6). In that sense we have observed that our approach allowed to produce results, which are more tailored to patients' needs. One may also notice, that the re-referral happens only when our approach is applied. This is due to the fact, that this activity is costly in terms of arrival time and as such is not favoured by the approaches, where the cost (time) criterion is of more importance.

The weights that we applied for solving the multi-criteria weighted sum aggregation (6.7) produced very similar results as opposed to each other. This is not an extensive list of weights and one should note that choosing different ones might produce different results. Yet, the assignment of weights is a difficult task and is of little applicability in the optimisation of dispatch of the EMS/ED services, where decisions must be made quickly and reliably.

6.4.5 Scenario 2

This Section gives numerical results of the test applied to Scenario 2, where direct transport to referential hospitals is not possible. In that sense, patients requiring specialised treatment will need to be first admitted to a non-referential ED and only then re-referred to a referential unit. This is a special scenario developed to analyse the impact of re-referrals applied instead of direct transport to referential hospitals. Results obtained are shown in Tab. 6.6.

Table 6.6: ED Dispatching Problem results (S2)

ED Dispatching Problem (P2) Scenario 2 (S2)					
Index	(6.3)	(6.6)	(6.7) $v_1 = 1$, $v_2 = 1$	(6.7) $v_1 = 0.8$, $v_2 = 15$	(6.7) $v_1 = 15$, $v_2 = 0.8$,
1	70.00	54.00	54.00	60.00	54.00
2	12.00	28.00	28.00	22.00	28.00
3	48.57%	77.50%	77.50%	77.50%	77.50%
4	−132.61	−566.35	−566.35	−494.71	−566.35
5	0.56	0.56	0.56	0.56	0.56
6	17.64	17.64	17.64	17.64	17.64
7	0.35	0.18	0.18	0.18	0.18
8	36.96	28.99	28.99	28.99	28.99
9	0.00	0.00	0.00	0.00	0.00
10	7.00	0.00	0.00	4.00	0.00

As can be seen, forbidding direct transfers to referential units worsened the overall indices (no. 1 to 4) for all of the approaches shown. This is mostly due to the fact that re-referral is often costly in terms of time, and therefore the optimiser would opt for sacrificing the speciality in order to meet patients' time requirements.

Thus, it is valid to conclude that re-referring is often not the best strategy, and therefore, the dispatcher should always consider patients' condition in the ED dispatching process to correctly direct the ambulance at the very moment of starting to transport the patient. Direct transports of patients to referential hospitals may greatly improve the performance of the EMS/ED services.

6.4.6 Scenario 3

Scenario 3 is a case in which re-referral is much less time-consuming than in normal operating conditions (S1); namely, it is one-fourth of the re-referral time from S1. The purpose of testing under this scenario is to check if the proposed approach might be interesting to the dispatcher if re-referrals were not problematic from a time perspective.

The results are shown in Tab. 6.7. As can be noted, from solving the proposed P2: ED Dispatching Problem, many more re-referrals were obtained, which improved the overall results. However, changing the re-referral times did not change the results of other approaches considered. One may then conclude that the P2 approach adapts itself better to changing decision environments from the approaches considered while still being viable.

Table 6.7: ED Dispatching Problem results (S3)

ED Dispatching Problem (P2) Scenario 3 (S3)					
Index	(6.3)	(6.6)	(6.7) $v_1 = 1$, $v_2 = 1$	(6.7) $v_1 = 0.8$, $v_2 = 15$	(6.7) $v_1 = 15$, $v_2 = 0.8$,
1	78.00	58.00	58.00	62.00	58.00
2	3.00	24.00	24.00	20.00	24.00
3	42.94%	77.50%	77.50%	77.50%	77.50%
4	−30.03	−470.23	−470.23	−404.35	−470.23
5	0.56	0.56	0.56	0.56	0.56
6	17.64	17.64	17.64	17.64	17.64
7	0.35	0.18	0.18	0.18	0.18
8	36.96	28.99	28.99	28.99	28.99
9	2.00	10.00	10.00	10.00	10.00
10	8.00	0.00	0.00	0.00	0.00

6.5 Summary

The main goal of any system of Emergency Medical Service is to provide timely and accurate medical support to patients in acute (often even life-threatening) conditions. It is the task of the emergency medical dispatcher (together with the chief of the emergency crew) to correctly dispatch EMS crews (ambulances) to patients and patients onboard the ambulances to Emergency Departments in hospitals. This task is currently facilitated by the use of call triage and categorisations systems (e.g., MPDS) and by means of patient transport protocols. The condition of the patients varies greatly amongst them (basing on their clinical condition), and so varies the speciality in treating given kinds of diseases between emergency system components. This is why treating all patients alike in the dispatch process, regardless of their clinical condition, is not a desired assignment strategy. In some cases providing medical aid with the wrong level of speciality might make the treatment less effective or even impossible.

In this Chapter, we applied our proposed generic multi-criteria Services of General Interest dispatching optimisation problem with quality-based criteria to operating the system of Emergency Medical Services (EMS) with Emergency Departments (EDs). By doing so, we aimed to develop a decision-support tool to be used by an emergency medical dispatcher in the dispatch process. The problems proposed consider patients' requirements towards both time of getting medical support and the speciality level of this support, which is a realisation of the quality criterion proposed in the generic problem. The requirements towards the criteria are expressed by means of aspiration and reservation values and assessed basing on patients' health conditions. We propose that this assessment is performed by properly trained medical personnel with the help of the currently used MPDS system, and by integrating current transport protocols. Time and speciality requirements are not uniform across acute-state patients and depend greatly on their medical condition. We take this fact into consideration in our optimisation problems by optimising for both time and speciality requirements of each patient individually (on a per-patient basis), where aspirations/reservations calculated depend on their clinical condition. In that sense, we can conclude, that the results obtained by applying our method are more tailored to patients needs and always consider the current operational state of the EMS system.

The proposed ED dispatching problem allows for optimising the decisions on whether to transport a given patient to a non-referential facility or directly to a referential one or to re-transfer them from a non-referential to a referential one. The decision is proposed by the optimiser, taking into consideration all patients' clinical conditions and the current operational state of the whole EMS system (availability of ambulances and hospital beds, speciality offered by available units, as well as time to reach the patient/ED). Thanks to optimising decisions considering this level of flexibility, one can suspect that wide adoption of the proposed system could possibly reduce offload delays in all types of hospitals. This is because patients will be transferred to destination hospitals basing on their clinical needs. Therefore, it is likely that patients not requiring specialist care will be directed to non-referential units and those requiring it to the referential ones. Re-transferring will be done only when critically required. All of the above take into consideration the current hospital capabilities and delays.

In this work, we also proposed an integration framework of the proposed decision support tools into the current EMS/ED dispatching decision process, which outlines integration with already existing dispatch tools. It was depicted schematically in the flow diagram The use of our method can potentially improve the performance of currently used techniques. Once the calls are categorised and triaged (the job of MPDS), the method allows for identifying and assigning the most appropriate unit to work with a given patient, considering the current operational state of the system as a whole. Similarly, the method can improve the use of current transport protocols by applying their guidelines in optimising for the best ED (in terms of its address) to admit a given patient (in terms of exact hospital location), also considering current hospital capabilities. The approach is tested in simulations using real-life emergency cases stored in the NEMSIS data set over different decision environment scenarios.

In all scenarios tested, the proposed approach managed to find a dispatch that is more tailored to the patients' needs than the other approaches shown. This is measured by the number of criteria being at least as good as their reservations associated, the number of criteria being worse from their reservations by at least 10%, the maximum percentage gap between the criterion value and its reservation, as well as the mean value of the utility function of the patients.

This research also has some limitations. First, we focused mostly on cardiological diseases for testing purposes. Yet, the problems proposed are generic enough that the type of disease could easily be changed to any other. What is more, the problems could also be extended to introduce other speciality measures towards other types of EMS service. These are, however, considered out of the scope of this Chapter.

As already stated, the method proposed is built on a per-patient basis and not on a per-incident basis. A question may then arise on how to handle the EMS assignment for incidents with multiple patients. Our method is also capable of handling multiple patients in one event. When dealing with an accident with multiple patients, each one of them should be identified as needing help. And for each one of them, we would have the time and speciality criterion assigned, with aspirations/reservations to each of them. In that sense, the dispatch of resources would be still in-line with the tailored-for-needs approach.

In case a mass event is present, it could simply not be possible to assign appropriate aspirations or reservations for the EMS speciality needed for each of the patients. In such a case we suggest to set the value of aspiration for speciality to 0.5 (middle of the range). Then, to assign reservation for the same to 0 (lower bound). However, we require that EMS arrive quickly and thus aspiration and reservation for time-to-arrival should be strict, e.g., reservation: 12 min, aspiration: 7 min. (depending on the current operational state of the system, and on medical protocols). In that way, the optimiser would aim to assign a unit, that can arrive at the scene as soon as possible, with a slight preference towards more specialised ones.

Secondly, as in our approach the dispatching is not based solely on the time criterion, EMS quality assurance key performance indicators should possibly be adjusted. Considering a certain percentage of calls to be served within a nationally defined time threshold might no longer be appropriate. We propose to measure the EMS performance by the percentage of all criteria (time/speciality) which are at least as good as their reservation and compare it with a derived threshold. This however is to be applied at a legislative level.

In the optimisation problems, we do not consider queuing, assuming that always at least as many EMS/ED units are available as the number of patients they should serve. This assumption does reflect the reality of some EMS systems. According to the Warsaw Office of Statistics (*Urzad Statystyczny w Warszawie*), in 2021 on average, only around 24% of available EMS units were busy per hour in the whole of Mazowieckie voivodeship, Poland [114]. This number is similar for other years too. However, acknowledging the fact that queuing models may add value to the problems developed in this study, we envisage considering them in future research. Another research possibility is to apply the approach proposed to real-life emergency medical system dispatching.

Despite the limitations, the application of the developed generic multi-criteria Services of General Interest dispatching optimisation problem with quality-based criteria proves to be an interesting dispatch strategy also for EMS/ED dispatch. The addition of quality-based criteria enables better differentiation of patients basing on their medical condition. This allows to better distribute the limited EMS/ED resources in order to better suit patients' needs. What is more, the proposed approach allows for consideration of both direct transports of patients to referential hospitals and re-referrals from non-referential units. In that sense, the approach gives more flexibility and allows for broader optimisation of dispatch decisions, as well as allows for getting dispatch results which are more tailored to patients' needs. All things considered, such an approach might possibly enhance patients' survival rate in emergencies.

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Chapter 7

Conclusions and discussion

This Chapter presents general conclusions drawn from this study. Specific conclusion Sections to other parts of this study were already given in the previous Chapters. We organise this Chapter as follows: first, we summarise the validation of the main research thesis. Then, we present the discussion on the limitations of this study and outline some future research possibilities.

7.1 Research outcomes

Services of General Interest are an important pillar of the social model and of the social market economy of the European Union. These services are focused on guaranteeing the minimum well-being standards for the Europeans so to make their lives better and less dependent on external factors. They may be constituted of service with either dispatchable or non-dispatchable units.

In this work, we aimed to contribute to the general research on Decision Support Systems (DSSs) for the dispatchers of the dispatchable SGIs. We considered only the SGIs, whose units may be differentiated based on the quality (or speciality) of the service offered. We wanted to verify the main research thesis as whether *adding quality-based criteria to the SGI dispatch optimisation problem adds value to the dispatching, allowing to yield more tailored results to customers' needs.* For this, we specifically developed the generic multi-criteria SGI dispatch optimisation problem with quality-based criteria (SDOQ), where not only does the optimiser focus on minimising the overall cost/time of the dispatch, but also simultaneously on assigning the appropriate service unit as required by the dispatch participants. Our ambition was to allow for producing research results which could improve the dispatch strategies considered currently in the DSSs for SGI dispatching, so that they suggest the most appropriate unit for a given customer. Here, we understand the appropriateness as how well the service dispatched responds to the requirements of the customers, which were established with domain-specific expertise and knowledge.

We verified the main research thesis, together with the performance of the proposed SDOQ problem in multiple steps. Firstly, we reviewed theoretically the characteristics of the problem and identified the Reference Point Method as the best suited aggregation method (Chapter 3). Once established, we tested it statistically on 6,000 artificial instances with randomly generated parameters (Chapter 4). It is worth mentioning here, that we performed the tests on the non-equitable and equitable (fair) formulations of a specific realisation of the SDOQ problem. Then, to make the investigations more realistic, we applied the problem SDOQ to two Case Studies from highly distinct fields. The first one was related to the dispatch of electrical energy generating units and the second one to the dispatch of Emergency Medical Services with Emergency Departments (Chapters 5 and 6). In both Case Studies we developed comprehensive mathematical models of the specific decision situations and performed some numerical experiments.

In all of the above-mentioned numerical tests we compared the performance of the SDOQ, as opposed to the standard single-criterion dispatch cost minimisation approaches. For that we looked at a number of different performance measures (indices), namely:

1. Number of criteria being at least as good as their reservations associated;
2. Number of criteria not meeting their reservations by at least 10%;
3. Maximum strictly positive percentage gap between the criterion value and its reservation, for both cost and quality criteria jointly;
4. Mean value of the utility function;
5. Optimiser's solution time (not investigated in the Case Studies);
6. Obtained values for cost and quality criteria by two selected consumers — no. 3 and no. 9 (only in the Case Studies).

All of our tests outlined that the overall appropriateness indices (no. 1 to 4) were always better when the multi-criteria approach with quality-based criteria was applied, both for the tests from Chapter 4 and the domain-specific Case Studies. It is especially worth noting that the mean value of the utility function over all participants was always significantly higher when the quality-based approach was applied. It is interesting, as this index represents the level of satisfaction of an average participant on the dispatch results. The indices considered measure, from multiple points of view, how appropriately the dispatcher is able to dispatch the SGI resources to their customers. It is specifically important as generally the SGI resources are limited and shall only be assigned in a way that they are able to serve appropriately as many customers as possible.

The improvement of the appropriateness indices came however, at the cost of significantly worsening the Optimiser's solution time with the increase of the problem size/complexity. The increase was especially visible while solving the equitable formulation of the SDOQ — making it difficult to be solved in reasonable time for SGI dispatching on our test laptop. However, this increase was less important when the non-equitable formulation was applied. In such case it was possible to solve the SDOQ problem in the time acceptable for the dispatching process of many real-life SGIs.

The Case Studies, which were based on our published papers [13, 63] have outlined that the SDOQ problem can be well applied to the dispatch of various SGI services, from very distinct fields. In each of those applications, the use of SDOQ allowed for yielding dispatch results, which were more tailored to customers' needs. It was of specific interest, as both EMS/ED and energy generation systems can be considered critical. Hence, a more appropriate dispatch of their limited resources is vital to the health and well-being of the Europeans.

All things considered, we have shown that the proposed SDOQ problem can work and bring value to multiple various and distinct applications of the Services of General Interest. Despite the fact that it solves in longer time, we can conclude that adding quality-based criteria to the generic Services of General Interest dispatch optimisation problem does add value to the dispatching, allowing to yield more tailored results to the customers' needs. As such, we conclude that the main research thesis has been confirmed in the course of the study presented.

7.2 Limitations of the study and research possibilities

This study was limited by the fact that it was conducted only in simulations. Although we based the Case Studies either on standard, widely known test cases, or on real-life data, we did not apply the results to real-life decision situations. What is more, the study was also limited by the fact of using artificially generated data sets in Chapter 4. It is recommended to perform further studies of the method proposed in real-life environments before applying it widely to the dispatching of various Services of General Interest. For instance, one may envisage conducting double-blind, randomised clinical trials for the dispatching of EMS services to patients in a real EMS system. A similar approach could be applied to other SGI dispatch optimisation problems — e.g., by creating a dedicated pilot of the proposed electrical energy market.

Secondly, we envisage the possibility of analysing different Case Studies — the list of possible candidates for applications is given in Sec. 1.1.5. Similar investigations as in Chapters 5 and 6 can be performed by proposing dedicated optimisation problems and testing them in appropriate simulations. Once successful, one can proceed to applying the results into real-life SGI dispatching situations.

Thirdly, in all problems, we have assumed that the number/amount of dispatchable units is at least as large as the number/amount of demand. In other words, we have not considered queuing. This was done on purpose — not to blur the answer to the main research question addressed in this study. Especially as this assumption is generally valid for the Case Studies we performed. What is more, the models developed in the course of this study were deterministic only. However, real-life situations are prone to uncertainties. Thus, another research possibility might include extending the models to more robust (or stochastic) formulations.

In the course of our analyses we were always happy with the first Pareto-optimal solution produced, as long as it responded to the customers' needs and requirements. We were motivated by the fact that in real-life situations SGI dispatcher's decisions must be made quickly, efficiently and correctly. Hence, we assumed that the first solution is valid, provided that it responds well to the requirements. However, detailed study on how many solutions should be presented to the dispatcher may be identified as a future research possibility.

Bibliography

[1] European Economic and Social Committee. Services of general interest. Accessed: 2022-06-05. URL: `https://www.eesc.europa.eu/en/policies/policy-areas/services-general-interest`.

[2] European Anti Poverty Network. Services of general interest: Glossary and terms explained. Technical report, 2007. Accessed: 2022-09-27.

[3] European Commission. *Guide to the application of the European Union rules on state aid, public procurement and the internal market to services of general economic interest, and in particular to social services of general interest*. Commission staff working document. 2013. Accessed: 2023-08-18. URL: `https://ec.europa.eu/competition/state_aid/overview/new_guide_eu_rules_procurement_en.pdf`.

[4] Jakub Kociubiński. Services of general economic interest – towards a european concept of public services. *Wroclaw Review of Law, Administration & Economics*, 1(2):49–64, 2011. `doi:10.2478/wrlae-2013-0027`.

[5] Lei Zhu. *Services of General Economic Interest in EU Competition Law*. T.M.C. Asser Press, The Hague, 2020. `doi:10.1007/978-94-6265-387-0`.

[6] European Commission. *A Quality Framework for Services of General Interest in Europe*. Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of the Regions. 2011. Accessed: 2023-08-18. URL: `https://ec.europa.eu/archives/commission_2010-2014/president/news/speeches-statements/pdf/20111220_1_en.pdf`.

[7] SGI Europe. Position Paper — The EU Single Market at 30: SGIs in the Single Market, 2023. Accessed: 2023-07-20. URL: `https://sgieurope.org/files/SGI%20Europe_Position%20Paper%20-%20SGIs%20in%20the%20Single%20Market_ok.pdf`.

[8] Dispatch; Cambridge University Press & Assessment. Cambridge Dictionary, 2023. Accessed: 2023-08-19. URL: https://dictionary.cambridge.org/dictionary/english/dispatch.

[9] Lingyu Zhang et al. A taxi order dispatch model based on combinatorial optimization. In *Proceedings of the 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, KDD '17, page 2151–2159, New York, NY, USA, 2017. Association for Computing Machinery. doi:10.1145/3097983.3098138.

[10] A.S. Carvalho, M.E. Captivo, and I. Marques. Integrating the ambulance dispatching and relocation problems to maximize system's preparedness. *European Journal of Operational Research*, 283(3):1064–1080, 2020. doi:10.1016/j.ejor.2019.11.056.

[11] Oscar Danilo Montoya and Walter Gil-González. Dynamic active and reactive power compensation in distribution networks with batteries: A day-ahead economic dispatch approach. *Computers & Electrical Engineering*, 85:106710, 2020. doi:10.1016/j.compeleceng.2020.106710.

[12] Faegheh Moazeni and Javad Khazaei. Dynamic economic dispatch of islanded water-energy microgrids with smart building thermal energy management system. *Applied Energy*, 276:115422, 2020. doi:10.1016/j.apenergy.2020.115422.

[13] Mariusz Drabecki and Eugeniusz Toczyłowski. Multi-objective approach for managing uncertain delivery from renewable energy sources within a peer-to-peer energy balancing architecture. *Energies*, 15(3), 2022. doi:10.3390/en15030675.

[14] V. Bélanger, A. Ruiz, and P. Soriano. Recent optimization models and trends in location, relocation, and dispatching of emergency medical vehicles. *European Journal of Operational Research*, 272(1):1–23, 2019. doi:10.1016/j.ejor.2018.02.055.

[15] Marc E. Pfetsch et al. Validation of nominations in gas network optimization: models, methods, and solutions. *Optimization Methods and Software*, 30(1):15–53, 2015. doi:10.1080/10556788.2014.888426.

[16] M. Münster, H. Ravn, K. Hedegaard, N. Juul, and M. Ljunggren Söderman. Economic and environmental optimization of waste treatment. *Waste Management*, 38:486–495, 2015. doi:10.1016/j.wasman.2014.12.005.

[17] Włodzimierz Ogryczak, Hanan Luss, Michał Pióro, Dritan Nace, and Artur Tomaszewski. Fair optimization and networks: A survey. *Journal of Applied Mathematics*, 2014, 2014. doi:10.1155/2014/612018.

[18] Xuefeng Li, Mingyang Du, and Mei-Po Kwan. Order dispatch optimization with considering flexible one-to-three matching strategy under hybrid ride-hailing service modes. *Transportation Letters*, 15(10):1337–1353, 2023. doi:10.1080/19427867.2022.2150705.

[19] Vilas Nitivattananon, Elaine C. Sadowski, and Rafael G. Quimpo. Optimization of water supply system operation. *Journal of Water Resources Planning and Management*, 122(5):374–384, 1996. doi:10.1061/(ASCE)0733-9496(1996)122:5(374).

[20] Mariusz Drabecki. A method for ensuring technical feasibility of distributed balancing in power systems, considering peer-to-peer balancing energy trade. In Pandian Vasant, Ivan Zelinka, and Gerhard-Wilhelm Weber, editors, *Intelligent Computing and Optimization*, pages 118–131, Cham, 2021. Springer International Publishing. doi:10.1007/978-3-030-68154-8_13.

[21] Mariusz Drabecki. A method for enhancing power system's steady-state voltage stability level by considering active power optimal dispatch with linear grid models. *Zeszyty Naukowe Wydziału Elektrotechniki i Automatyki Politechniki Gdańskiej*, 62:55–58, 2019. doi:10.32016/1.62.10.

[22] Mariusz Drabecki and Eugeniusz Toczyłowski. Obtaining feasibility of power flows in the deregulated electricity market environment. *Przegląd Elektrotechniczny*, 95(10):44–47, 2019. doi:10.15199/48.2019.10.08.

[23] David Arnott and Graham Pervan. Eight key issues for the decision support systems discipline. *Decision Support Systems*, 44(3):657–672, 2008. doi:10.1016/j.dss.2007.09.003.

[24] G. Anthony Gorry and Michael S. Scott Morton. A framework for management information systems. *Sloan Management Review pre-1986)*, 13(1):55, 1971.

[25] Daniel J. Power. Decision support systems: A historical overview. In *Handbook on Decision Support Systems 1: Basic Themes*, pages 121–140. Springer Berlin Heidelberg, Berlin, Heidelberg, 2008. doi:10.1007/978-3-540-48713-5_7.

[26] Andreas Felsberger, Bernhard Oberegger, and Gerald Reiner. A review of decision support systems for manufacturing systems. *SAMI@ iKNOW*, page 8, 2016.

[27] Azad M. Madni, Carla C. Madni, and Scott D. Lucero. Leveraging digital twin technology in model-based systems engineering. *Systems*, 7(1), 2019. doi:10.3390/systems7010007.

[28] NoorUl Ain, Giovanni Vaia, William H. DeLone, and Mehwish Waheed. Two decades of research on business intelligence system adoption, utilization and success – a systematic literature review. *Decision Support Systems*, 125:113113, 2019. doi:10.1016/j.dss.2019.113113.

[29] Janusz Kacprzyk and Sławomir Zadrożny. Fuzzy linguistic summaries in text categorization for human-consistent document-driven decision support systems. In Bernd Reusch, editor, *Computational Intelligence, Theory and Applications*, pages 271–279, Berlin, Heidelberg, 2005. Springer Berlin Heidelberg. doi:10.1007/3-540-31182-3_24.

[30] Reyes Michaela Denise Gonzales and Carol Anne Hargreaves. How can we use artificial intelligence for stock recommendation and risk management? A proposed decision support system. *International Journal of Information Management Data Insights*, 2(2):100130, 2022. doi:10.1016/j.jjimei.2022.100130.

[31] Foivos Psarommatis and Dimitris Kiritsis. A hybrid decision support system for automating decision making in the event of defects in the era of zero defect manufacturing. *Journal of Industrial Information Integration*, 26:100263, 2022. doi:10.1016/j.jii.2021.100263.

[32] Guanghui Zhou, Chao Zhang, Zhi Li, Kai Ding, and Chuang Wang. Knowledge-driven digital twin manufacturing cell towards intelligent manufacturing. *International Journal of Production Research*, 58(4):1034–1051, 2020. doi:10.1080/00207543.2019.1607978.

[33] Czesław Smutnicki. Discrete optimization in the industrial computer science. In Piotr Kulczycki, Józef Korbicz, and Janusz Kacprzyk, editors, *Automatic Control, Robotics, and Information Processing*, pages 359–385. Springer International Publishing, Cham, 2021. doi:10.1007/978-3-030-48587-0_12.

[34] Clyde W. Holsapple. DSS architecture and types. In *Handbook on Decision Support Systems 1: Basic Themes*, pages 163–189. Springer Berlin Heidelberg, Berlin, Heidelberg, 2008. doi:10.1007/978-3-540-48713-5_9.

[35] Wojtek Michalowski, Steven Rubin, Roman Słowiński, and Szymon Wilk. Mobile clinical support system for pediatric emergencies. *Decision Support Systems*, 36(2):161–176, 2003. doi:10.1016/S0167-9236(02)00140-9.

[36] Cambridge University Press & Assessment. Quality; Cambridge Dictionary, 2023. Accessed: 2023-07-14. URL: https://dictionary.cambridge.org/dictionary/english/quality.

[37] Mathias H.J. Bollen. What is power quality? *Electric Power Systems Research*, 66(1):5–14, 2003. Power Quality. doi:10.1016/S0378-7796(03)00067-1.

[38] Choton K. Das et al. Optimal allocation of distributed energy storage systems to improve performance and power quality of distribution networks. *Applied Energy*, 252:113468, 2019. doi:10.1016/j.apenergy.2019.113468.

[39] Robert Denda, Albert Banchs, and Wolfgang Effelsberg. The fairness challenge in computer networks. In Jon Crowcroft, James Roberts, and Mikhail I. Smirnov, editors, *Quality of Future Internet Services*, pages 208–220, Berlin, Heidelberg, 2000. Springer Berlin Heidelberg. doi:10.1007/3-540-39939-9_17.

[40] The International Telegraph and Telephone Consultative Committee. *Recommendation E.800: Quality of Service and Dependability Vocabulary*. Series E: Overall Network Operation, Telephone Service, Service Operation and Human Factors. International Telecommunication Union, 1988.

[41] Vasileios Karagiannis, Periklis Chatzimisios, Francisco Vazquez-Gallego, and Jesus Alonso-Zarate. A survey on application layer protocols for the internet of things. *Transaction on IoT and Cloud computing*, 3(1):11–17, 2015. doi:10.5281/zenodo.51613.

[42] Joanna Józefowska, Mariusz Nowak, Rafał Różycki, and Grzegorz Waligóra. Survey on optimization models for energy-efficient computing systems. *Energies*, 15(22), 2022. doi:10.3390/en15228710.

[43] Abderrezak Rachedi and Abderrahim Benslimane. Multi-objective optimization for security and QoS adaptation in wireless sensor networks. In *2016 IEEE International Conference on Communications (ICC)*, pages 1–7, 2016. doi:10.1109/ICC.2016.7510879.

[44] Neda Mazloomi, Majid Gholipour, and Arash Zaretalab. Efficient configuration for multi-objective QoS optimization in wireless sensor network. *Ad Hoc Networks*, 125:102730, 2022. doi:10.1016/j.adhoc.2021.102730.

[45] Tadeusz Sałaciński. *Quality Engineering in Manufacturing Technology*. Oficyna Wydawnicza Politechniki Warszawskiej, 1 edition, 2020.

[46] Jizhong Zhu. *Optimization of power system operation*. John Wiley & Sons, Hoboken, New Jersey, 2015. doi:10.1002/9781118887004.

[47] Eugeniusz Toczyłowski. *Optymalizacja procesów rynkowych przy ograniczeniach [Optimization of market processes under constraints]*. Problemy Współczesnej Nauki, Teoria

i Zastosowania. Zarządzanie i Informatyka [Problems of current research, theory and applications]. Akademicka Oficyna Wydawnicza Exit, Warszawa, 2 edition, 2003.

[48] Melanie Reuter-Oppermann, Pieter L. van den Berg, and Julie L. Vile. Logistics for Emergency Medical Service systems. *Health Systems*, 6(3):187–208, 2017. doi:10.1057/s41306-017-0023-x.

[49] Raimund Erbel et al. 2014 ESC Guidelines on the diagnosis and treatment of aortic diseases: Document covering acute and chronic aortic diseases of the thoracic and abdominal aorta of the adult The Task Force for the Diagnosis and Treatment of Aortic Diseases of the European Society of Cardiology (ESC). *European Heart Journal*, 35(41):2873–2926, 08 2014. doi:10.1093/eurheartj/ehu281.

[50] Lumír Francek, Ota Hlinomaz, Ladislav Groch, and Silvie Belaskova. Analysis of time intervals related to stemi management in 2008-2016. *Cor et Vasa*, 60(3):e239–e245, 2018. doi:10.1016/j.crvasa.2017.07.007.

[51] Stavros V. Konstantinides et al. 2019 ESC Guidelines for the diagnosis and management of acute pulmonary embolism developed in collaboration with the European Respiratory Society (ERS). *European Respiratory Journal*, 54(3), 2019. doi:10.1183/13993003.01647-2019.

[52] Mattias Vesterlund, Andrea Toffolo, and Jan Dahl. Optimization of multi-source complex district heating network, a case study. *Energy*, 126:53–63, 2017. doi:10.1016/j.energy.2017.03.018.

[53] Elias Yaacoub, Zaher Dawy, et al. Fair optimization of video streaming quality of experience in lte networks using distributed antenna systems and radio resource management. *Journal of Applied Mathematics*, 2014, 2014. doi:10.1155/2014/562079.

[54] Haotong Cao, Sahil Garg, Georges Kaddoum, Mohammad Mehedi Hassan, and Salman A. AlQahtani. Intelligent virtual resource allocation of QoS-guaranteed slices in B5G-Enabled VANETs for intelligent transportation systems. *IEEE Transactions on Intelligent Transportation Systems*, 23(10):19704–19713, 2022. doi:10.1109/TITS.2022.3178267.

[55] City of Temple Terrace. The Divisions and Structure of the Police Department. Accessed: 2023-04-15. URL: https://www.templeterrace.gov/308/Divisions.

[56] Jimmy J. Williams and David Westall. SWAT and non-SWAT police officers and the use of force. *Journal of Criminal Justice*, 31(5):469–474, 2003. doi:10.1016/S0047-2352(03)00051-5.

[57] Jan Terpstra. Local Policing in a Nationalized Police Force: A Study on the Local Teams of The Netherlands' National Police. *Policing: A Journal of Policy and Practice*, 15(1):251–262, 06 2018. doi:10.1093/police/pay037.

[58] Komenda Główna Państwowej Straży Pożarnej. Organizacja KSRG [Organisation of the Polish Fire Rescue Service]. Accessed: 2023-04-15. URL: https://www.gov.pl/web/kgpsp/organizacja-ksrg.

[59] Renata Albergaria de Mello Bandeira, George Vasconcelos Goes, Daniel Neves Schmitz Gonçalves, Márcio de Almeida D'Agosto, and Cíntia Machado de Oliveira. Electric vehicles in the last mile of urban freight transportation: A sustainability assessment of postal deliveries in Rio de Janeiro — Brazil. *Transportation Research Part D: Transport and Environment*, 67:491–502, 2019. doi:10.1016/j.trd.2018.12.017.

[60] Jorgen Aarhaug. Taxis as part of public transport. Technical report, German Federal Ministry of Economic Cooperation and Development, 2016.

[61] Uber. Selecting a vehicle option for your ride. Accessed: 2023-04-15. URL: https://help.uber.com/riders/article/selecting-a-vehicle-option-for-your-ride?nodeId=53d5ef29-0fa5-4252-a48c-43ff244ce6ce.

[62] Free Now. Can I order a more environmentally friendly vehicle? Accessed: 2023-04-15. URL: https://support.free-now.com/hc/en-gb/articles/360015210179-Can-I-order-a-more-environmentally-friendly-vehicle-.

[63] Mariusz Drabecki, Eugeniusz Toczyłowski, Krzysztof Pieńkosz, Grzegorz Honisz, and Klaudia Kułak. Multi-criteria assignment problems for optimising the emergency medical services (EMS), considering non-homogeneous speciality of the emergency departments and EMS crews. *Scientific Reports*, 13(1):1–19, 2023. doi:10.1038/s41598-023-33831-7.

[64] Temel Öncan. A survey of the generalized assignment problem and its applications. *INFOR: Information Systems and Operational Research*, 45(3):123–141, 2007. doi:10.3138/infor.45.3.123.

[65] Nancy V. Phillips. A weighting function for pre-emptive multicriteria assignment problems. *Journal of the Operational Research Society*, 38:797–802, 1987. doi:10.1057/jors.1987.133.

[66] Ekunda Lukata Ulungu and Jacques Teghem. Multi-objective combinatorial optimization problems: A survey. *Journal of Multi-Criteria Decision Analysis*, 3(2):83–104, 1994. doi:10.1002/mcda.4020030204.

[67] Esmaiel Keshavarz and Mehdi Toloo. Finding efficient assignments: An innovative DEA approach. *Measurement*, 58:448–458, 2014. doi:10.1016/j.measurement.2014.09.014.

[68] Włodzimierz Ogryczak. *Wielokryterialna Optymalizacja Liniowa i Dyskretna - modele preferencji i zastosowania do wspomagania decyzji [Multicriteria linear and discrete optimisation — preference models and applications to decision support]*. Uniwersytet Warszawski [University of Warsaw], 1997.

[69] Matthias Ehrgott. A discussion of scalarization techniques for multiple objective integer programming. *Annals of Operations Research*, 147(1):343–360, 2006. doi:10.1007/s10479-006-0074-z.

[70] Gholam Hassan Shirdel and Ameneh Mortezaee. A DEA-based approach for the multi-criteria assignment problem. *Croatian Operational Research Review*, 6(1):145–154, 2015. doi:10.17535/crorr.2015.0012.

[71] Lyes Belhoul, Lucie Galand, and Daniel Vanderpooten. An efficient procedure for finding best compromise solutions to the multi-objective assignment problem. *Computers & Operations Research*, 49:97–106, 2014. doi:10.1016/j.cor.2014.03.016.

[72] Anthony Przybylski, Xavier Gandibleux, and Matthias Ehrgott. A two phase method for multi-objective integer programming and its application to the assignment problem with three objectives. *Discrete Optimization*, 7(3):149–165, 2010. doi:10.1016/j.disopt.2010.03.005.

[73] Shanu Verma, Millie Pant, and Vaclav Snasel. A comprehensive review on NSGA-II for multi-objective combinatorial optimization problems. *IEEE Access*, 9:57757–57791, 2021. doi:10.1109/ACCESS.2021.3070634.

[74] Qi Liu, Xiaofeng Li, Haitao Liu, and Zhaoxia Guo. Multi-objective metaheuristics for discrete optimization problems: A review of the state-of-the-art. *Applied Soft Computing*, 93:106382, 2020. doi:10.1016/j.asoc.2020.106382.

[75] Gülesin Sena Daş, Fatma Gzara, and Thomas Stützle. A review on airport gate assignment problems: Single versus multi objective approaches. *Omega*, 92:102146, 2020. doi:10.1016/j.omega.2019.102146.

[76] Ignacy Kaliszewski, Janusz Miroforidis, and Jarosław Tomasz Stańczak. Multiobjective optimization in the airport gate assignment problem, exact versus evolutionary multiobjective optimization. *Computer Science*, 18(1):41, 2017. doi:10.7494/csci.2017.18.1.41.

[77] Seray Çakırgil, Eda Yücel, and Gültekin Kuyzu. An integrated solution approach for multi-objective, multi-skill workforce scheduling and routing problems. *Computers & Operations Research*, 118:104908, 2020. doi:10.1016/j.cor.2020.104908.

[78] Niteesh Yadav and Ajinkya Tanksale. A multi-objective approach for reducing patient's inconvenience in a generalized home healthcare delivery setup. *Expert Systems with Applications*, 219:119657, 2023. doi:10.1016/j.eswa.2023.119657.

[79] Matthew Thomas Fletcher. *Balancing Talent and Cost:Multi-Objective Optimization of US Army Officer Assignments*. PhD thesis, North Carolina State University, 2021.

[80] Matthew D. Ferguson, Raymond Hill, and Brian Lunday. A scenario-based parametric analysis of the army personnel-to-assignment matching problem. *Journal of Defense Analytics and Logistics*, 4(1):89–106, 2020. doi:10.1108/jdal-08-2019-0015.

[81] Bartosz Sawik. Application of multi-criteria mathematical programming models for assignment of services in a hospital. *Applications of management science*, 16:39–53, 2013. doi:10.1108/S0276-8976(2013)0000016006.

[82] Bartosz Sawik. Lexicographic approach to health care optimization. *Applications of management science*, 17:181–201, 2015. doi:10.1108/S0276-897620140000170130.

[83] Bartosz Sawik. Multi-criteria mathematical programming approaches for assignment of services in hospital. In *International Conference on Industrial Logistics — ICIL 2016*, pages 239–246, 2016. URL: http://www.icil.zarz.agh.edu.pl/images/papers/SawikB.pdf.

[84] Byung Duk Song, Hyorin Park, and Kyungsu Park. Toward flexible and persistent UAV service: Multi-period and multi-objective system design with task assignment for disaster management. *Expert Systems with Applications*, 206:117855, 2022. doi:10.1016/j.eswa.2022.117855.

[85] Xiaohua Gao et al. Conditional probability based multi-objective cooperative task assignment for heterogeneous UAVs. *Engineering Applications of Artificial Intelligence*, 123:106404, 2023. doi:10.1016/j.engappai.2023.106404.

[86] Hao-Xiang Chen, Ying Nan, and Yi Yang. Multi-UAV reconnaissance task assignment for heterogeneous targets based on modified symbiotic organisms search algorithm. *Sensors*, 19(3), 2019. doi:10.3390/s19030734.

[87] Yu Qiu, Hong Jiang, Qingdong Li, Xiwang Dong, and Zhang Ren. Application of an adapted genetic algorithm on task allocation problem of multiple UAVs. In *2018 IEEE CSAA Guidance, Navigation and Control Conference (CGNCC)*, pages 1–6, 2018. doi:10.1109/GNCC42960.2018.9018985.

[88] Guodong Lyu, Wang Chi Cheung, Chung-Piaw Teo, and Hai Wang. Multi-objective stochastic optimization: A case of real-time matching in ride-sourcing markets. *Available at SSRN*, 3356823:21, 2019. doi:10.2139/ssrn.3356823.

[89] Jianguo Chen, Kenli Li, Keqin Li, Philip S Yu, and Zeng Zeng. Dynamic bicycle dispatching of dockless public bicycle-sharing systems using multi-objective reinforcement learning. *ACM Transactions on Cyber-Physical Systems (TCPS)*, 5(4):1–24, 2021. doi:10.1145/3447623.

[90] Tadeusz Sawik. A linear model for optimal cybersecurity investment in industry 4.0 supply chains. *International Journal of Production Research*, 60(4):1368–1385, 2022. doi:10.1080/00207543.2020.1856442.

[91] Gerald F. Reid and Lawrence Hasdorff. Economic dispatch using quadratic programming. *IEEE Transactions on Power Apparatus and Systems*, PAS-92(6):2015–2023, 1973. doi:10.1109/TPAS.1973.293582.

[92] Carlos Henggeler Antunes and Carla Oliveira Henriques. Multi-objective optimization and multi-criteria analysis models and methods for problems in the energy sector. In Salvatore Greco, Matthias Ehrgott, and Jose Rui Figueira, editors, *Multiple Criteria Decision Analysis: State of the Art Surveys*, pages 1067–1165. Springer New York, New York, NY, 2016. doi:10.1007/978-1-4939-3094-4_25.

[93] B.Y. Qu, Y.S. Zhu, Y.C. Jiao, M.Y. Wu, P.N. Suganthan, and J.J. Liang. A survey on multi-objective evolutionary algorithms for the solution of the environmental/economic dispatch problems. *Swarm and Evolutionary Computation*, 38:1–11, 2018. doi:10.1016/j.swevo.2017.06.002.

[94] Abimbola M. Jubril, Olusola A. Komolafe, and Kehinde O. Alawode. Solving multi-objective economic dispatch problem via semidefinite programming. *IEEE Transactions on Power Systems*, 28(3):2056–2064, 2013. doi:10.1109/TPWRS.2013.2245688.

[95] Hui Hou et al. Multi-objective economic dispatch of a microgrid considering electric vehicle and transferable load. *Applied Energy*, 262:114489, 2020. doi:10.1016/j.apenergy.2020.114489.

[96] Haitao Liu, Yu Ji, Huaidong Zhuang, and Hongbin Wu. Multi-objective dynamic economic dispatch of microgrid systems including vehicle-to-grid. *Energies*, 8(5):4476–4495, 2015. doi:10.3390/en8054476.

[97] Teodoro Cardoso Bora, Viviana Cocco Mariani, and Leandro dos Santos Coelho. Multi-objective optimization of the environmental-economic dispatch with reinforcement learning based on non-dominated sorting genetic algorithm. *Applied Thermal Engineering*, 146:688–700, 2019. doi:10.1016/j.applthermaleng.2018.10.020.

[98] Yixuan Chen and Yunhe Hou. Fast yet balanced trade-offs for multi-timescale multi-objective economic-environmental dispatch under varying conflicts. *Applied Energy*, 328:120122, 2022. doi:10.1016/j.apenergy.2022.120122.

[99] Masoud Dashtdar et al. Solving the environmental/economic dispatch problem using the hybrid FA-GA multi-objective algorithm. *Energy Reports*, 8:13766–13779, 2022. doi:10.1016/j.egyr.2022.10.054.

[100] Sardar Ansari, Soovin Yoon, and Laura A. Albert. An approximate hypercube model for public service systems with co-located servers and multiple response. *Transportation Research Part E: Logistics and Transportation Review*, 103:143–157, 2017. doi:10.1016/j.tre.2017.04.013.

[101] Ricardo Franceli da Silva et al. Tariff optimization under incentive regulation of public services using simplex method. *Procedia Computer Science*, 214:1325–1333, 2022. 9th International Conference on Information Technology and Quantitative Management. doi:10.1016/j.procs.2022.11.312.

[102] Vladimir Marianov and Daniel Serra. Location problems in the public sector. In Zvi Drezner and Horst W. Hamacher, editors, *Facility location: Applications and theory*, pages 119–150. Springer, Berlin, Heidelberg, 2002.

[103] Arthur J. Swersey. The deployment of police, fire, and emergency medical units. In *Operations Research and The Public Sector*, volume 6 of *Handbooks in Operations Research and Management Science*, pages 151–200. Elsevier, 1994. doi:10.1016/S0927-0507(05)80087-8.

[104] Po-Lung Yu. *Multiple-Criteria Decision Making: Concepts, Techniques, and Extensions*, volume 30 of *Mathematical Concepts and Methods in Science and Engineering*. Springer, Boston, MA, 1985. doi:10.1007/978-1-4684-8395-6.

[105] Michael M. Kostreva, Włodzimierz Ogryczak, and Adam Wierzbicki. Equitable aggregations and multiple criteria analysis. *European Journal of Operational Research*, 158(2):362–377, 2004. Methodological Foundations of Multi-Criteria Decision Making. doi:10.1016/j.ejor.2003.06.010.

[106] B.A. Ghaznavi-Ghosoni and E. Khorram. On approximating weakly/properly efficient solutions in multi-objective programming. *Mathematical and Computer Modelling*, 54(11):3172–3181, 2011. doi:10.1016/j.mcm.2011.08.013.

[107] Michael M. Kostreva and Włodzimierz Ogryczak. Linear optimization with multiple equitable criteria. *RAIRO-Operations Research-Recherche Operationnelle*, 33(3):275–297, 1999. doi:10.1051/ro:1999112.

[108] Vijay K. Singh. *Equitable efficiency in multiple criteria optimization*. PhD thesis, Clemson University, 2007.

[109] Erio Castagnoli and Pietro Muliere. A note on inequality measures and the Pigou-Dalton principle of transfers. In Camilo Dagum and Michele Zenga, editors, *Income and Wealth Distribution, Inequality and Poverty*, pages 171–182, Berlin, Heidelberg, 1990. Springer Berlin Heidelberg. doi:10.1007/978-3-642-84250-4_11.

[110] Włodzimierz Ogryczak, Adam Wierzbicki, and Marcin Milewski. A multi-criteria approach to fair and efficient bandwidth allocation. *Omega*, 36(3):451–463, 2008. doi:10.1016/j.omega.2005.12.005.

[111] Andrzej Lewandowski and Andrzej P. Wierzbicki. *Aspiration Based Decision Support Systems: Theory, Software and Applications*, volume 331 of *Lecture Notes in Economics and Mathematical Systems*. Springer Berlin / Heidelberg, Berlin, Heidelberg, 1989. doi:10.1007/978-3-662-21637-8.

[112] Peter J. Kolesar, Kenneth L. Rider, Thomas B. Crabill, and Warren E. Walker. A queuing-linear programming approach to scheduling police patrol cars. *Operations Research*, 23(6):1045–1062, 1975. doi:10.1287/opre.23.6.1045.

[113] Shakiba Enayati, Maria E. Mayorga, Hector Toro-Díaz, and Laura A. Albert. Identifying trade-offs in equity and efficiency for simultaneously optimizing location and multipriority dispatch of ambulances. *International Transactions in Operational Research*, 26(2):415–438, 2019. doi:10.1111/itor.12590.

[114] Urząd Statystyczny w Warszawie [Warsaw Statistics Office]. Ratownictwo medyczne w województwie mazowieckim w 2021 r. [Emergency Medical Services in Mazowieckie voivodeship in year 2021], 2022. URL: https://warszawa.stat.gov.pl/opracowania-biezace/opracowania-sygnalne/zdrowie-pomoc-spoleczna/ratownictwo-medyczne-w-wojewodztwie-mazowieckim-w-2021-r-,1,4.html.

[115] Gabriele Eichfelder. *Adaptive Scalarization Methods in Multiobjective Optimization*. Springer Berlin Heidelberg, Berlin, Heidelberg, 2008. doi:10.1007/978-3-540-79159-1_2.

[116] Pascal Halffmann, Luca E. Schäfer, Kerstin Dächert, Kathrin Klamroth, and Stefan Ruzika. Exact algorithms for multiobjective linear optimization problems with integer variables: A state of the art survey. *Journal of Multi-Criteria Decision Analysis*, 29(5-6):341–363, 2022. doi:10.1002/mcda.1780.

[117] Gurobi Optimization. Gurobi optimizer. Accessed: 12/02/2023. URL: https://www.gurobi.com/.

[118] IBM. ILOG CPLEX Optimizer. Accessed: 12/02/2023. URL: https://www.ibm.com/pl-pl/analytics/cplex-optimizer.

[119] MOSEK. Accessed: 12/02/2023. URL: https://mosek.com.

[120] FICO. Fico Xpress Solver. Accessed: 12/02/2023. URL: https://www.fico.com/en/products/fico-xpress-solver.

[121] IBM. Multiobjective optimization. Accessed: 12/02/2023. URL: https://www.ibm.com/docs/en/icos/12.10.0?topic=optimization-multiobjective.

[122] Gurobi Optimization. Working with multiple objectives. Accessed: 02/12/2023. URL: https://www.gurobi.com/documentation/current/refman/working_with_multiple_obje.htmle.

[123] FICO. Multi-objective optimization. Accessed: 02/12/2023. URL: https://www.fico.com/fico-xpress-optimization/docs/latest/solver/optimizer/HTML/chapter5.html?scroll=multiobj.

[124] Sebastian Schenker. *Multicriteria Linear Optimisation with Applications in Sustainable Manufacturing*. PhD thesis, Technische Universitaet Berlin (Germany), 2019.

[125] Ralf Borndörfer, Sebastian Schenker, Martin Skutella, and Timo Strunk. PolySCIP. In Gert-Martin Greuel, Thorsten Koch, Peter Paule, and Andrew Sommese, editors, *Math-*

ematical Software – ICMS 2016, pages 259–264, Cham, 2016. Springer International Publishing. doi:10.1007/978-3-319-42432-3_32.

[126] Andreas Löhne and Benjamin Weißing. The vector linear program solver BENSOLVE – notes on theoretical background. *European Journal of Operational Research*, 260(3):807–813, 2017. doi:10.1016/j.ejor.2016.02.039.

[127] Andreas Löhne and Benjamin Weißing. BENSOLVE — A Free Vector Linear Program Solver. Accessed: 12/02/2023. URL: http://www.bensolve.org/.

[128] László Csirmaz. inner. Accessed: 12/02/2023. URL: https://github.com/csirmaz/inner/blob/master/README.md.

[129] Ted K. Ralphs, Matthew J. Saltzman, and Margaret M. Wiecek. An improved algorithm for solving biobjective integer programs. *Annals of Operations Research*, 147:43–70, 2006. doi:10.1007/s10479-006-0058-z.

[130] Anthony Daniels. TridentOpt: A Multi-Objective Optimization Engine or Solver. Accessed: 12/02/2023. URL: https://www.codeproject.com/Articles/1214046/TridentOpt-A-Multi-bjective-Optimization-Engine-o.

[131] Matthias Ehrgott. *Multicriteria optimization*. Springer Berlin, Berlin, Heidelberg, 2005. doi:10.1007/978-3-662-22199-0.

[132] Ronald R. Yager. On ordered weighted averaging aggregation operators in multicriteria decisionmaking. *IEEE Transactions on Systems, Man, and Cybernetics*, 18(1):183–190, 1988. doi:10.1109/21.87068.

[133] Andrzej P. Wierzbicki. A mathematical basis for satisficing decision making. *Mathematical Modelling*, 3(5):391–405, 1982. doi:10.1016/0270-0255(82)90038-0.

[134] Andrzej P. Wierzbicki. On the completeness and constructiveness of parametric characterizations to vector optimization problems. *Operations-Research-Spektrum*, 8(2):73–87, 1986. doi:10.1007/BF01719738.

[135] Lech Kruś, Jan Skorupiński, and Eugeniusz Toczyłowski. Analysis of incentive compatible multicriteria decisions for a producer and buyers problem. *Multiple Criteria Decision Making*, 7:132–145, 2012.

[136] Lech Kruś and Eugeniusz Toczyłowski. Remarks on designing iterative multicriteria procurement auctions. *Multiple Criteria Decision Making*, 9:32–47, 2014.

[137] Włodzimierz Ogryczak. *Wspomaganie decyzji*. Politechnika Warszawska, Ośrodek Kształcenia na Odległość OKNO, 2006.

[138] Borja Ibanez and ESC Scientific Document Group. 2017 ESC Guidelines for the management of acute myocardial infarction in patients presenting with ST-segment elevation: The Task Force for the management of acute myocardial infarction in patients presenting with ST-segment elevation of the European Society of Cardiology (ESC). *European Heart Journal*, 39(2):119–177, 2017. doi:10.1093/eurheartj/ehx393.

[139] Włodzimierz Ogryczak and Bartosz Kozłowski. Reference point method with importance weighted ordered partial achievements. *Top*, 19:380–401, 2011. doi:10.1007/s11750-009-0121-4.

[140] James P. Ignizio. Generalized goal programming an overview. *Computers & Operations Research*, 10(4):277–289, 1983. doi:10.1016/0305-0548(83)90003-5.

[141] Dylan Jones and Mehrdad Tamiz. *Practical goal programming*. Springer New York, New York, NY, 2010. doi:10.1007/978-1-4419-5771-9.

[142] Włodzimierz Ogryczak. Preemptive reference point method. In João Clímaco, editor, *Multicriteria Analysis*, pages 156–167, Berlin, Heidelberg, 1997. Springer Berlin Heidelberg. doi:10.1007/978-3-642-60667-0_16.

[143] Yacov Y. Haimes, Leon S. Lasdon, and David A. Wismer. On a bicriterion formulation of the problems of integrated system identification and system optimization. *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-1(3):296–297, 1971. doi:10.1109/TSMC.1971.4308298.

[144] El-Ghazali Talbi, Matthieu Basseur, Antonio J. Nebro, and Enrique Alba. Multi-objective optimization using metaheuristics: non-standard algorithms. *International Transactions in Operational Research*, 19(1-2):283–305, 2012. doi:10.1111/j.1475-3995.2011.00808.x.

[145] Michael Grant and Stephen Boyd. CVX: Matlab software for disciplined convex programming, version 2.1, March 2014. URL: http://cvxr.com/cvx.

[146] Michael C. Grant and Stephen P. Boyd. Graph implementations for nonsmooth convex programs. In Vincent D. Blondel, Stephen P. Boyd, and Hidenori Kimura, editors, *Recent Advances in Learning and Control*, pages 95–110, London, 2008. Springer London. doi:10.1007/978-1-84800-155-8_7.

[147] Mathworks. MATLAB Documentation - filloutliers. Accessed: 16/02/2023. URL: https://www.mathworks.com/help/matlab/ref/filloutliers.html.

[148] Przemysław Grzegorzewski, Konstancja Bobecka, Anna Dembińska, and Jerzy Pusz. *Rachunek prawdopodobieństwa i statystyka [Theory of probability and statistics]*. Wyższa Szkoła Informatyki Stosowanej i Zarządzania, Warszawa, 5 edition, 2008.

[149] Mariusz Piotr Drabecki and Klaudia Brygida Kułak. Global pandemics on European electrical energy markets: Lessons learned from the COVID-19 outbreak. *International Journal of Energy Optimization and Engineering (IJEOE)*, 10(3):24–46, 2021. doi:10.4018/ijeoe.2021070102.

[150] Larry Wasserman. *All of statistics: a concise course in statistical inference*. Springer New York, NY, New York, 2004. doi:10.1007/978-0-387-21736-9.

[151] Yael Parag and Benjamin K. Sovacool. Electricity market design for the prosumer era. *Nature energy*, 1(4):1–6, 2016. doi:10.1038/nenergy.2016.32.

[152] Open Utility. A glimpse into the future of britain's energy economy, 2021. Accessed: 2021-11-11. URL: https://piclo.energy/publications/piclo-trial-report.pdf.

[153] Vanderbron. Vanderbron, 2021. Accessed: 2021-11-11. URL: https://vandebron.nl/.

[154] Sonnen Group. Sonnencommunity. https://sonnengroup.com/sonnencommunity/, 2021. Accessed: 2021-11-11.

[155] Accenture. How we'll get to 100% renewable energy. Accessed: 2021-11-11. URL: https://www.accenture.com/us-en/blogs/blogs-renewable-energy.

[156] Marcella Nicolini and Massimo Tavoni. Are renewable energy subsidies effective? Evidence from Europe. *Renewable and Sustainable Energy Reviews*, 74:412–423, 2017. doi:10.1016/j.rser.2016.12.032.

[157] Australian Renewable Energy Agency; AGL; IBM; MHC. Virtual peer-to-peer energy trading using distributed ledger technology: Comprehensive project assessment report, 2017. Accessed: 2021-11-11. URL: https://arena.gov.au/knowledge-bank/peer-peer-distributed-ledger-technology-assessment/.

[158] Wayes Tushar, Tapan Kumar Saha, Chau Yuen, David Smith, and H. Vincent Poor. Peer-to-peer trading in electricity networks: An overview. *IEEE Transactions on Smart Grid*, 11(4):3185–3200, 2020. doi:10.1109/TSG.2020.2969657.

[159] Omid Abrishambaf, Fernando Lezama, Pedro Faria, and Zita Vale. Towards transactive energy systems: An analysis on current trends. *Energy Strategy Reviews*, 26:100418, 2019. doi:10.1016/j.esr.2019.100418.

[160] Tiago Sousa, Tiago Soares, Pierre Pinson, Fabio Moret, Thomas Baroche, and Etienne Sorin. Peer-to-peer and community-based markets: A comprehensive review. *Renewable and Sustainable Energy Reviews*, 104:367–378, 2019. doi:10.1016/j.rser.2019.01.036.

[161] Jaysson Guerrero, Daniel Gebbran, Sleiman Mhanna, Archie C. Chapman, and Gregor Verbič. Towards a transactive energy system for integration of distributed energy resources: Home energy management, distributed optimal power flow, and peer-to-peer energy trading. *Renewable and Sustainable Energy Reviews*, 132:110000, 2020. doi:10.1016/j.rser.2020.110000.

[162] Esteban A. Soto, Lisa B. Bosman, Ebisa Wollega, and Walter D. Leon-Salas. Peer-to-peer energy trading: A review of the literature. *Applied Energy*, 283:116268, 2021. doi:10.1016/j.apenergy.2020.116268.

[163] Lurian Pires Klein, Aleksandra Krivoglazova, Luisa Matos, Jorge Landeck, and Manuel de Azevedo. A novel peer-to-peer energy sharing business model for the Portuguese energy market. *Energies*, 13(1), 2020. doi:10.3390/en13010125.

[164] Pablo Baez-Gonzalez, Enrique Rodriguez-Diaz, Juan C. Vasquez, and Josep M. Guerrero. Peer-to-peer energy market for community microgrids [technology leaders]. *IEEE Electrification Magazine*, 6(4):102–107, 2018. doi:10.1109/mele.2018.2871326.

[165] Muhammad Fahad Zia, Elhoussin Elbouchikhi, Mohamed Benbouzid, and Josep M. Guerrero. Microgrid transactive energy systems: A perspective on design, technologies, and energy markets. In *IECON 2019 — 45th Annual Conference of the IEEE Industrial Electronics Society*, volume 1, pages 5795–5800, 2019. doi:10.1109/iecon.2019.8926947.

[166] Thomas Morstyn, Niall Farrell, Sarah J. Darby, and Malcolm D. McCulloch. Using peer-to-peer energy-trading platforms to incentivize prosumers to form federated power plants. *Nature Energy*, 3(2):94–101, 2018. doi:10.1038/s41560-017-0075-y.

[167] Liliane Ableitner, Verena Tiefenbeck, Arne Meeuw, Anselma Wörner, Elgar Fleisch, and Felix Wortmann. User behavior in a real-world peer-to-peer electricity market. *Applied Energy*, 270:115061, 2020. doi:10.1016/j.apenergy.2020.115061.

[168] Chao Long, Jianzhong Wu, Chenghua Zhang, Meng Cheng, and Ali Al-Wakeel. Feasibility of peer-to-peer energy trading in low voltage electrical distribution networks. *Energy Procedia*, 105:2227–2232, 2017. doi:10.1016/j.egypro.2017.03.632.

[169] Chenghua Zhang, Jianzhong Wu, Yue Zhou, Meng Cheng, and Chao Long. Peer-to-peer energy trading in a microgrid. *Applied Energy*, 220:1–12, 2018. doi:10.1016/j.apenergy.2018.03.010.

[170] Hélène Le Cadre, Paulin Jacquot, Cheng Wan, and Clémence Alasseur. Peer-to-peer electricity market analysis: From variational to generalized Nash equilibrium. *European Journal of Operational Research*, 282(2):753–771, 2020. doi:10.1016/j.ejor.2019.09.035.

[171] Wayes Tushar et al. A motivational game-theoretic approach for peer-to-peer energy trading in the smart grid. *Applied Energy*, 243:10–20, 2019. doi:10.1016/j.apenergy.2019.03.111.

[172] Muhammad Raisul Alam, Marc St-Hilaire, and Thomas Kunz. An optimal p2p energy trading model for smart homes in the smart grid. *Energy Efficiency*, 10(6):1475–1493, 2017. doi:10.1007/s12053-017-9532-5.

[173] Muhammad Raisul Alam, Marc St-Hilaire, and Thomas Kunz. Peer-to-peer energy trading among smart homes. *Applied Energy*, 238:1434–1443, 2019. doi:10.1016/j.apenergy.2019.01.091.

[174] Jaysson Guerrero, Archie C. Chapman, and Gregor Verbič. Decentralized p2p energy trading under network constraints in a low-voltage network. *IEEE Transactions on Smart Grid*, 10(5):5163–5173, 2018. doi:10.1109/TSG.2018.2878445.

[175] M. S. H. Nizami, M. J. Hossain, B. M. Ruhul Amin, Muhammad Kashif, Edstan Fernandez, and Khizir Mahmud. Transactive energy trading of residential prosumers using battery energy storage systems. In *2019 IEEE Milan PowerTech*, pages 1–6, 2019. doi:10.1109/PTC.2019.8810458.

[176] Roberto Alvaro-Hermana, Jesus Fraile-Ardanuy, Pedro J Zufiria, Luk Knapen, and Davy Janssens. Peer to peer energy trading with electric vehicles. *IEEE Intelligent Transportation Systems Magazine*, 8(3):33–44, 2016. doi:10.1109/MITS.2016.2573178.

[177] Junjie Hu, Guangya Yang, Charalampos Ziras, and Koen Kok. Aggregator operation in the balancing market through network-constrained transactive energy. *IEEE Transactions on Power Systems*, 34(5):4071–4080, 2019. doi:10.1109/TPWRS.2018.2874255.

[178] C.K. Woo, P. Sreedharan, J. Hargreaves, F. Kahrl, J. Wang, and I. Horowitz. A review of electricity product differentiation. *Applied Energy*, 114:262–272, 2014. doi:10.1016/j.apenergy.2013.09.070.

[179] Etienne Sorin, Lucien Bobo, and Pierre Pinson. Consensus-based approach to peer-to-peer electricity markets with product differentiation. *IEEE Transactions on Power Systems*, 34(2):994–1004, 2019. doi:10.1109/TPWRS.2018.2872880.

[180] Tommaso Orlandini, Tiago Soares, Tiago Sousa, and Pierre Pinson. Coordinating consumer-centric market and grid operation on distribution grid. In *2019 16th International Conference on the European Energy Market (EEM)*, pages 1–6, 2019. doi:10.1109/EEM.2019.8916247.

[181] Tauron Dystrybucja. Instrukcja Ruchu i Eksploatacji Sieci Dystrybucyjnej [*Distribution Grid Code*]. Accessed: 2021-11-11. URL: https://www.tauron-dystrybucja.pl/uslugi-dystrybucyjne/instrukcja-iriesd.

[182] Mariusz Drabecki and Eugeniusz Toczyłowski. Comparison of three approaches to the security constrained unit commitment problem. *Zeszyty Naukowe Wydziału Elektrotechniki i Automatyki Politechniki Gdańskiej*, 62:67–70, 2019. doi:10.32016/1.62.13.

[183] Allen J. Wood, Bruce F. Wollenberg, and Gerald B. Sheblé. *Power Generation, Operation, and Control (3rd Edition)*. John Wiley & Sons, 2014.

[184] Fangxing Li and Rui Bo. DCOPF-based LMP simulation: Algorithm, comparison with ACOPF, and sensitivity. *IEEE Transactions on Power Systems*, 22(4):1475–1485, 2007. doi:10.1109/TPWRS.2007.907924.

[185] Anastasios G. Bakirtzis and Pandelis N. Biskas. A decentralized solution to the DC-OPF of interconnected power systems. *IEEE Transactions on Power Systems*, 18(3):1007–1013, 2003. doi:10.1109/TPWRS.2003.814853.

[186] Mariusz Drabecki. Optimization methods for voltage stability enhancement in electric power dispatch problem. Master's thesis, Warsaw University of Technology, Warsaw, Poland, 2018.

[187] University of Washington. IEEE 30-bus system. Accessed: 2021-10-09. URL: https://labs.ece.uw.edu/pstca/pf30/pg_tca30bus.htm.

[188] Ray Daniel Zimmerman, Carlos Edmundo Murillo-Sánchez, and Robert John Thomas. Matpower: Steady-state operations, planning, and analysis tools for power systems research and education. *IEEE Transactions on Power Systems*, 26(1):12–19, 2011. doi:10.1109/TPWRS.2010.2051168.

[189] Solcast, 2019. global solar irradiance data and pv system power output data. Accessed: 2021-10-09. URL: https://solcast.com.

[190] Abdel-Fattah Attia, Ragab A. El Sehiemy, and Hany M. Hasanien. Optimal power flow solution in power systems using a novel sine-cosine algorithm. *International Journal of Electrical Power & Energy Systems*, 99:331–343, 2018. doi:10.1016/j.ijepes.2018.01.024.

[191] Seokcheon Lee. A new preparedness policy for EMS logistics. *Health care management science*, 20:105–114, 2017. doi:10.1007/s10729-015-9340-4.

[192] Grace M. Carter, Jan M. Chaiken, and Edward Ignall. Response areas for two emergency units. *Operations Research*, 20(3):571–594, 1972. doi:10.1287/opre.20.3.571.

[193] Verena Schmid. Solving the dynamic ambulance relocation and dispatching problem using approximate dynamic programming. *European Journal of Operational Research*, 219(3):611–621, 2012. doi:10.1016/j.ejor.2011.10.043.

[194] Caroline J. Jagtenberg, Sandjai Bhulai, and Robert D. van der Mei. Dynamic ambulance dispatching: is the closest-idle policy always optimal? *Health care management science*, 20(4):517–531, 2017. doi:10.1007/s10729-016-9368-0.

[195] Ustawa z dnia 8 września 2006 r. o Państwowym Ratownictwie Medycznym [The Act on the State National Emergency System], 2006.

[196] EENA - European Emergency Number Association. Emergency call handling service chain description v 2.0, 2020. URL: https://eena.org/wp-content/uploads/2020_12_07_ServiceChainV2.0-final.pdf.

[197] Joanna Sowizdraniuk et al. ECG pre-hospital teletransmission by emergency teams staffed with an emergency physician and paramedics and its impact on transportation and hospital admission. *Medicine*, 98(34), 2019. doi:10.1097/MD.0000000000016636.

[198] Peter J. H. Hulshof, Nikky Kortbeek, Richard J. Boucherie, Erwin W. Hans, and Piet J. M. Bakker. Taxonomic classification of planning decisions in health care: a structured review of the state of the art in OR/MS. *Health systems*, 1(2):129–175, 2012. doi:10.1057/hs.2012.18.

[199] Klaudiusz Nadolny and Jerzy Robert Ładny. Analiza interwencji medycznych u pacjentów w stanie zagrożenia życia — wybrane stany nagłe [Analysis of medical interventions in patients in life-threatening conditions — selected emergency medical conditions]. *Pielęgniarstwo w stanach nagłych*, 2, 2018.

[200] Piotr Gajewski, editor. *Interna Szczeklika: maly podrecznik 2018/2019 [Szczeklik's internal medicine: little handbook 2018/2019]*. Medycyna Praktyczna, Kraków, 2018.

[201] Brian A. Bergmark, Njambi Mathenge, Piera A. Merlini, Marilyn B. Lawrence-Wright, and Robert P. Giugliano. Acute coronary syndromes. *The Lancet*, 399(10332):1347–1358, 2022. doi:10.1016/S0140-6736(21)02391-6.

[202] Narodowy Fundusz Zdrowia [Polish National Health Fund]. *NFZ o zdrowiu. Choroba niedokrwienna serca [NFZ on health. Ischemic heart disease]*. URL: https://ezdrowie.gov.pl/portal/home/badania-i-dane/zdrowe-dane/raporty/nfz-o-zdrowiu-choroba-niedokrwienna-serca.

[203] Michal Feliks Czapla. *Powiklania w transportach medycznych u pacjentów z ostrym zespolem wieńcowym wykonywanych przez specjalistyczne oraz podstawowe zespoły ratownictwa medycznego [Complications in medical transport in patients with acute coronary syndrome performed by specialised and basic EMS crews]*. PhD thesis, Wrocław Medical University, 2017.

[204] Jarosław Gucwa and Maciej Ostrowski. *Zaawansowane zabiegi resuscytacyjne i wybrane stany nagle [Advanced life support and selected emergencies]*. Medycyna Praktyczna, Kraków, 2015.

[205] Dariusz Dudek et al. Organizacja interwencyjnego leczenia pacjentów z zawałem serca STEMI i NSTEMI w Polsce [Invasive treatment strategies in patients with myocardial infarction in Poland]. *Kardiologia Polska*, 68(5), 2010.

[206] Sang-Beom Jeon et al. Neurological emergencies in patients hospitalized with nonneurological illness. *Journal of Patient Safety*, 17(8):e1332–e1340, 2021. doi:10.1097/PTS.0000000000000682.

[207] Wenxiu He, Zhiwen Zhang, and Xianyi Sha. Nanoparticles-mediated emerging approaches for effective treatment of ischemic stroke. *Biomaterials*, 277:121111, 2021. doi:10.1016/j.biomaterials.2021.121111.

[208] William J. Powers et al. Guidelines for the early management of patients with acute ischemic stroke: 2019 update to the 2018 guidelines for the early management of acute ischemic stroke: a guideline for healthcare professionals from the American Heart Association/American Stroke Association. *Stroke*, 50(12):e344–e418, 2019. doi:10.1161/STR.0000000000000211.

[209] Wiktor Szczuchniak, Piotr Sobolewski, and Grzegorz Kozera. Opóźnienia przed-i wewnatrzszpitalne w udarze mózgu: przyczyny, skutki, zapobieganie [Pre- and in-hospital delays in stroke: backgrounds, results and prevention]. *Forum Medycyny Rodzinnej*, 10(3):119–128, 2016.

[210] Frank J. Wolters, Nicola L. M. Paul, Linxin Li, and Peter M. Rothwell. Sustained impact of UK FAST-test public education on response to stroke: a population-based time-series study. *International Journal of Stroke*, 10(7):1108–1114, 2015. doi:10.1111/ijs.12484.

[211] Matthew S. W. Milne et al. Drip 'n ship versus mothership for endovascular treatment: modeling the best transportation options for optimal outcomes. *Stroke*, 48(3):791–794, 2017. doi:10.1161/STROKEAHA.116.015321.

[212] Intercollegiate Stroke Working Party. *National clinical guideline for stroke*. Royal College of Physicians, London, 2012.

[213] Federico Semeraro et al. European Resuscitation Council Guidelines 2021: Systems saving lives. *Resuscitation*, 161:80–97, 2021. doi:10.1016/j.resuscitation.2021.02.008.

[214] Alabama Department of Public Health. Alabama EMS patient care protocols, 10th Edition, 2022. URL: https://www.alabamapublichealth.gov/ems/assets/10th.edition.protocols.pdf.

[215] Delaware Office of Emergency Medical Services. Delaware Advanced Life Support Protocols,Guidelines, Policies and Standing Orders, 2022. URL: https://www.dhss.delaware.gov/dhss/dph/ems/files/ALSFinal2022.pdf.

[216] Marlena Robakowska, Anna Tyrańska-Fobke, Daniel Ślęzak, Michał Rogowski, Andrzej Basiński, and Sylwia Jałtuszewska. Command support system EMS (SWD PRM). *Emergency Medical Service*, 8(3):126–129, 2021. doi:10.36740/EmeMS202103102.

[217] Sergey V. Kovalchuk, Evgeniy Krotov, Pavel A. Smirnov, Denis A. Nasonov, and Alexey N. Yakovlev. Distributed data-driven platform for urgent decision making in cardiological ambulance control. *Future Generation Computer Systems*, 79:144–154, 2018. doi:10.1016/j.future.2016.09.017.

[218] R. Aringhieri, M.E. Bruni, S. Khodaparasti, and J.T. van Essen. Emergency medical services and beyond: Addressing new challenges through a wide literature review. *Computers & Operations Research*, 78:349–368, 2017. doi:10.1016/j.cor.2016.09.016.

[219] Kanglin Liu, Qiaofeng Li, and Zhi-Hai Zhang. Distributionally robust optimization of an emergency medical service station location and sizing problem with joint chance constraints. *Transportation Research Part B: Methodological*, 119:79–101, 2019. doi:10.1016/j.trb.2018.11.012.

[220] Roberto Aringhieri, Giuliana Carello, and Daniela Morale. Supporting decision making to improve the performance of an Italian Emergency Medical Service. *Annals of operations research*, 236(1):131–148, 2016. doi:10.1007/s10479-013-1487-0.

[221] Caroline J. Jagtenberg and Andrew J. Mason. Improving fairness in ambulance planning by time sharing. *European Journal of Operational Research*, 280(3):1095–1107, 2020. doi:10.1016/j.ejor.2019.08.003.

[222] Juan Carlos Dibene, Yazmin Maldonado, Carlos Vera, Mauricio de Oliveira, Leonardo Trujillo, and Oliver Schütze. Optimizing the location of ambulances in Tijuana, Mexico. *Computers in Biology and Medicine*, 80:107–115, 2017. doi:10.1016/j.compbiomed.2016.11.016.

[223] Hector Toro-Díaz, Maria E. Mayorga, Laura A. McLay, Hari K. Rajagopalan, and Cem Saydam. Reducing disparities in large-scale emergency medical service systems. *Journal of the Operational Research Society*, 66(7):1169–1181, 2015. doi:10.1057/jors.2014.83.

[224] Vidya K. Sudarshan, Mikkel Brabrand, Troels Martin Range, and Uffe Kock Wiil. Performance evaluation of Emergency Department patient arrivals forecasting models by including meteorological and calendar information: A comparative study. *Computers in Biology and Medicine*, 135:104541, 2021. doi:10.1016/j.compbiomed.2021.104541.

[225] Carla Taramasco, Rodrigo Olivares, Roberto Munoz, Ricardo Soto, Matías Villar, and Victor Hugo C. de Albuquerque. The patient bed assignment problem solved by autonomous bat algorithm. *Applied Soft Computing*, 81:105484, 2019. doi:10.1016/j.asoc.2019.105484.

[226] Soroush Saghafian, Garrett Austin, and Stephen J. Traub. Operations research/management contributions to emergency department patient flow optimization: Review and research prospects. *IIE Transactions on Healthcare Systems Engineering*, 5(2):101–123, 2015. doi:10.1080/19488300.2015.1017676.

[227] Rym Ben Bachouch, Alain Guinet, and Sonia Hajri-Gabouj. An integer linear model for hospital bed planning. *International Journal of Production Economics*, 140(2):833–843, 2012. doi:10.1016/j.ijpe.2012.07.023.

[228] Caroline J. Jagtenberg, Pieter L. van den Berg, and Robert D. van der Mei. Benchmarking online dispatch algorithms for emergency medical services. *European Journal of Operational Research*, 258(2):715–725, 2017. doi:10.1016/j.ejor.2016.08.061.

[229] Laura A. McLay and Maria E. Mayorga. A dispatching model for server-to-customer systems that balances efficiency and equity. *Manufacturing & Service Operations Management*, 15(2):205–220, 2013. doi:10.1287/msom.1120.0411.

[230] Cheng Siong Lim, Rosbi Mamat, and Thomas Braunl. Impact of ambulance dispatch policies on performance of emergency medical services. *IEEE Transactions on Intelligent Transportation Systems*, 12(2):624–632, 2011. doi:10.1109/TITS.2010.2101063.

[231] Justin J. Boutilier and Timothy C. Y. Chan. Ambulance emergency response optimization in developing countries. *Operations Research*, 68(5):1315–1334, 2020. doi:10.1287/opre.2019.1969.

[232] Kanchala Sudtachat, Maria E. Mayorga, and Laura A. McLay. Recommendations for dispatching emergency vehicles under multitiered response via simulation. *International Transactions in Operational Research*, 21(4):581–617, 2014. doi:10.1111/itor.12083.

[233] Pedro Marinho Sizenando Silva and Luiz Ricardo Pinto. Emergency medical systems analysis by simulation and optimization. In *Proceedings of the 2010 Winter Simulation Conference*, pages 2422–2432, 2010. doi:10.1109/WSC.2010.5678938.

[234] Ali Haghani, Huijun Hu, and Qiang Tian. An optimization model for real-time emergency vehicle dispatching and routing. In *82nd annual meeting of the Transportation Research Board, Washington, DC*. Transportation Research Board, 2003.

[235] Vincent A. Knight, Paul R. Harper, and Leanne Smith. Ambulance allocation for maximal survival with heterogeneous outcome measures. *Omega*, 40(6):918–926, 2012. doi:10.1016/j.omega.2012.02.003.

[236] Jack Stout, Paul E. Pepe, and Vincent N. Mosesso. All-advanced life support vs tiered-response ambulance systems. *Prehospital Emergency Care*, 4(1):1–6, 2000. doi:10.1080/10903120090941542.

[237] Wojtek Michalowski, Roman Słowiński, and Szymon Wilk. MET system: a new approach to m-health in emergency triage. In *Medical and Care Compunetics 1*, pages 101–108. IOS Press, 2004. doi:10.3233/978-1-60750-946-2-101.

[238] Karl A. Sporer and Nicholas J. Johnson. Detailed analysis of prehospital interventions in medical priority dispatch system determinants. *Western Journal of Emergency Medicine*, 12(1):19, 2011.

[239] Matthias Fischer et al. Comparing emergency medical service systems—A project of the European Emergency Data (EED) Project. *Resuscitation*, 82(3):285–293, 2011. doi:10.1016/j.resuscitation.2010.11.001.

[240] Karl A. Sporer, Glen M. Youngblood, and Robert M. Rodriguez. The ability of emergency medical dispatch codes of medical complaints to predict ALS prehospital interventions. *Prehospital emergency care*, 11(2):192–198, 2007. doi:10.1080/10903120701205984.

[241] Justin Yap et al. Performance of the medical priority dispatch system in correctly classifying out-of-hospital cardiac arrests as appropriate for resuscitation. *Resuscitation*, 181:123–131, 2022. doi:10.1016/j.resuscitation.2022.11.001.

[242] Xuejie Dong et al. Optimizing an emergency medical dispatch system to improve prehospital diagnosis and treatment of acute coronary syndrome: Nationwide retrospective study in China. *Journal of Medical Internet Research*, 24(11):e36929, 2022. doi:10.2196/36929.

[243] Emily Andrew et al. Aligning ambulance dispatch priority to patient acuity: a methodology. *Emergency Medicine Australasia*, 31(3):405–410, 2019. doi:10.1111/1742-6723.13181.

[244] Luca Talarico, Frank Meisel, and Kenneth Sörensen. Ambulance routing for disaster response with patient groups. *Computers & Operations Research*, 56:120–133, 2015. doi:10.1016/j.cor.2014.11.006.

[245] Edoardo Amaldi, Stefano Coniglio, and Claudio Iuliano. Optimization models for injured people evacuation in medium/maxi health-care emergencies. In *2010 IEEE Workshop on Health Care Management (WHCM)*, pages 1–6, 2010. doi:10.1109/WHCM.2010.5441250.

[246] Jorge A. Acuna, José L. Zayas-Castro, and Hadi Charkhgard. Ambulance allocation optimization model for the overcrowding problem in US emergency departments: A case study in Florida. *Socio-Economic Planning Sciences*, 71:100747, 2020. doi:10.1016/j.seps.2019.100747.

[247] Gianmaria Leo, Andrea Lodi, Paolo Tubertini, and Mirko Di Martino. Emergency department management in Lazio, Italy. *Omega*, 58:128–138, 2016. doi:10.1016/j.omega.2015.05.007.

[248] Urząd Wojewódzki w Kielcach. Plan działania systemu Państwowe Ratownictwo Medyczne dla województwa świętokrzyskiego [Plan of operating the National Emergency Medical Service in the Świętokrzyskie Voivodeship], 2022. URL: https://czkw.kielce.uw.gov.pl/download/4/32964/WPDSPRM-wojswietokrzyskieaktualizacjaNr7BIP.pdf.

[249] Douglas F. Kupas, Ellen Schenk, J. Matthew Sholl, and Richard Kamin. Characteristics of statewide protocols for emergency medical services in the United States. *Prehospital Emergency Care*, 19(2):292–301, 2015. doi:10.3109/10903127.2014.964891.

[250] Carlin C. Chuck et al. Statewide Emergency Medical Services protocols for suspected stroke and large vessel occlusion. *JAMA neurology*, 78(11):1404–1406, 2021. doi:10.1001/jamaneurol.2021.3227.

[251] Magdalena Żurowska Wolak, Patryk Piekos, Jacek Jąkała, and Marcin Mikos. The effects of prehospital system delays on the treatment efficacy of STEMI patients. *Scandinavian journal of trauma, resuscitation and emergency medicine*, 27(39):1–7, 2019. doi:10.1186/s13049-019-0616-4.

[252] Peter Clemmensen et al. Pre-hospital diagnosis and transfer of patients with acute myocardial infarction — a decade long experience from one of Europe's largest STEMI networks. *Journal of Electrocardiology*, 46(6):546–552, 2013. doi:10.1016/j.jelectrocard.2013.07.004.

[253] Janke S. Wiegersma, Joep M. Droogh, Jan G. Zijlstra, Janneke Fokkema, and Jack J. M. Ligtenberg. Quality of interhospital transport of the critically ill: impact of a mobile intensive care unit with a specialized retrieval team. *Critical care*, 15(R75):1–8, 2011. doi:10.1186/cc10064.

[254] Ewa Kaniecka et al. Evaluation of selected items of emergency medical system in Poland by practitioners. *Emergency Medical Service*, 7(2):113–119, 2020. doi:10.36740/EmeMS202002106.

[255] National Highway Traffic Safety Administration (NHTSA), Office of Emergency Medical Services (OEMS). 2020 National Emergency Medical Services Information System (NEMSIS) Public-Release Research Dataset, 2020. URL: https://nemsis.org.

[256] National Highway Traffic Safety Administration (NHTSA), Office of Emergency Medical Services (OEMS). NEMSIS Data Dictionary Version 3.5.0, 2021. URL: https://nemsis.org/media/nemsis_v3/release-3.5.0/DataDictionary/PDFHTML/EMSDEMSTATE_National/NEMSISDataDictionary.pdf.

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Appendix A

Binary relations

In multi-criteria analysis, relations play a vital role. In this Section, we give some definitions related to them, which will be later referred to. This is cited after [108].

Definition A.0.1 (Binary relation)**.** Let $\mathcal{S}$ be a set. A *binary relation* on $\mathcal{S}$ is a subset R of $\mathcal{S} \times \mathcal{S}$, where $\mathcal{S} \times \mathcal{S} = \{(s^1, s^2) \mid s^1, s^2 \in \mathcal{S}\}$. If $(s^1, s^2) \in R$, we write $s^1 R s^2$.

Important qualities of binary relations, which are used in this work, are given below. Other qualities, which are not directly referred to in the remainder of the text of this work, are omitted, despite them being important for the theory of multi-criteria optimisation. However, to get more grasp of those, a reader is encouraged to consult [108] for more information.

A binary relation R is called:

1. *Reflexive*, if $sRs \; \forall s \in \mathcal{S}$
2. *Transitive*, if $s^1 R s^2$ and $s^2 R s^3 \implies s^1 R s^3 \; \forall s^1, s^2, s^3 \in \mathcal{S}$

Appendix B

Formulations for numerical experiments

The formulation (B.1) presents one of the optimisation problems used for the numerical experiments from the Chapter 4. It aggregates the multi-criteria problem in a way that it minimises the total sum of all cost criteria, summed over all customers. The quality criteria are not subjected to optimisation in the formulation. Therefore, we understand it as the cost-minimal approach. It represents the most classical dispatching policy, where the least expensive units are always deployed. In the case of ambulance dispatch, it recommends sending always the closest idle unit and in the case of generation of the electrical energy — currently only cheapest units/technologies. We refer to it in Chapter 4 as the *single-criterion* problem.

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{|\mathcal{P}_k|} \sum_{k=1}^{|\mathcal{I}|} c_{i,k} \\
\text{s.t.} \quad & q_{i,n} = f^1_{i,n}(\boldsymbol{x}) \qquad \forall i = 1, \ldots, |\mathcal{P}_k| \quad \forall n = 1, \ldots, |\mathcal{I}|, \\
& c_{i,k} = f^2_{i,k}(\boldsymbol{x}) \qquad \forall i = 1, \ldots, |\mathcal{P}_k| \quad \forall k = 1, \ldots, |\mathcal{K}|, \\
& f^1_{i,n}(\boldsymbol{x}) = \sum_{j=1}^{|\mathcal{P}_s|} y^j_i \; d^j_{i,n} \qquad \forall i = 1, \ldots, |\mathcal{P}_k| \quad \forall n = 1, \ldots, |\mathcal{I}|, \\
& f^2_{i,k}(\boldsymbol{x}) = \sum_{j=1}^{|\mathcal{P}_s|} y^j_i \; t^j_{i,k} \qquad \forall i = 1, \ldots, |\mathcal{P}_k| \quad \forall k = 1, \ldots, |\mathcal{K}|, \\
& \sum_{j=1}^{|\mathcal{P}_s|} y^j_i = 1 \qquad \forall i = 1, \ldots, |\mathcal{P}_k|, \\
& y^j_i \in \{0, 1\} \qquad \forall i = 1, \ldots, |\mathcal{P}_k| \quad \forall j = 1, \ldots, |\mathcal{P}_s|
\end{aligned}
\tag{B.1}
$$

The problem (B.2) is the direct formulation of the Reference Point Method scalarisation of the test formulation (4.1) from the Chapter 4. It optimises for both quality and cost criteria considering participants' preferences/requirements. Those are given in the form of aspirations and reservations $\boldsymbol{a}, \boldsymbol{r}$ towards all criteria. Each participant has its aspiration and reservation on each of their associated criteria. This problem does not ensure the equity of the result, but only its Pareto-optimality. It is referred to in Chapter 4 as the *multi-criteria non-equitable* problem.

$$\begin{aligned}
\max \quad & v + \rho \sum_{i=1}^{|\mathcal{P}_k|} \sum_{n=1}^{|\mathcal{I}|} \sum_{k=1}^{|\mathcal{K}|} (z^q_{i,n} + z^c_{i,k}) \\
\text{s.t.} \quad & q_{i,n} = f^1_{i,n}(\boldsymbol{x}) \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall n = 1,\dots,|\mathcal{I}|, \\
& c_{i,k} = f^2_{i,k}(\boldsymbol{x}) \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall k = 1,\dots,|\mathcal{K}|, \\
& f^1_{i,n}(\boldsymbol{x}) = \sum_{j=1}^{|\mathcal{P}_s|} y^j_i \, d^j_{i,n} \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall n = 1,\dots,|\mathcal{I}|, \\
& f^2_{i,k}(\boldsymbol{x}) = \sum_{j=1}^{|\mathcal{P}_s|} y^j_i \, t^j_{i,k} \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall k = 1,\dots,|\mathcal{K}|, \\
& \sum_{j=1}^{|\mathcal{P}_s|} y^j_i = 1 \qquad \forall i = 1,\dots,|\mathcal{P}_k|, \\
& y^j_i \in \{0,1\} \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall j = 1,\dots,|\mathcal{P}_s|, \\
& v \le z_w \qquad \forall w = 1,\dots,|\mathcal{P}_k| + |\mathcal{I}| + |\mathcal{K}|, \\
& z^q_{i,n} \le \gamma \frac{q_{i,n} - r^q_{i,n}}{a^q_{i,n} - r^q_{i,n}} \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall n = 1,\dots,|\mathcal{I}|, \\
& z^q_{i,n} \le \frac{q_{i,n} - r^q_{i,n}}{a^q_{i,n} - r^q_{i,n}} \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall n = 1,\dots,|\mathcal{I}|, \\
& z^q_{i,n} \le \beta \frac{q_{i,n} - a^q_{i,n}}{a^q_{i,n} - r^q_{i,n}} + 1 \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall n = 1,\dots,|\mathcal{I}|, \\
& z^c_{i,k} \le \gamma \frac{c_{i,k} - r^c_{i,k}}{a^c_{i,k} - r^c_{i,k}} \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall k = 1,\dots,|\mathcal{K}|, \\
& z^c_{i,k} \le \frac{c_{i,k} - r^c_{i,k}}{a^c_{i,k} - r^c_{i,k}} \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall k = 1,\dots,|\mathcal{K}|, \\
& z^c_{i,k} \le \beta \frac{c_{i,k} - a^c_{i,k}}{a^c_{i,k} - r^c_{i,k}} + 1 \qquad \forall i = 1,\dots,|\mathcal{P}_k| \quad \forall k = 1,\dots,|\mathcal{K}|
\end{aligned} \tag{B.2}$$

The problem (B.3) is an RPM scalarisation of (4.1) considering both equity and Pareto-optimality of the result. It optimises all criteria considering participants' preferences using the implementable version of the Nucleolar RPM, as given in (3.9). It is referred to in Chapter 4 as the *multi-criteria equitable* problem.

$$
\begin{aligned}
\max\quad & v + \rho \sum_{o=1}^{|\mathcal{P}_k|+|\mathcal{I}|+|\mathcal{K}|} g_o \\
\text{s.t.}\quad & q_{i,n} = f^1_{i,n}(\boldsymbol{x}) \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall n = 1,\ldots,|\mathcal{I}|, \\
& c_{i,k} = f^2_{i,k}(\boldsymbol{x}) \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall k = 1,\ldots,|\mathcal{K}|, \\
& f^1_{i,n}(\boldsymbol{x}) = \sum_{j=1}^{|\mathcal{P}_s|} y_i^j \, d^j_{i,n} \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall n = 1,\ldots,|\mathcal{I}|, \\
& f^2_{i,k}(\boldsymbol{x}) = \sum_{j=1}^{|\mathcal{P}_s|} y_i^j \, t^j_{i,k} \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall k = 1,\ldots,|\mathcal{K}|, \\
& \sum_{j=1}^{|\mathcal{P}_s|} y_i^j = 1 \qquad \forall i = 1,\ldots,|\mathcal{P}_k|, \\
& y_i^j \in \{0,1\} \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall j = 1,\ldots,|\mathcal{P}_s|, \\
& z^q_{i,n} \le \gamma \frac{q_{i,n} - r^q_{i,n}}{a^q_{i,n} - r^q_{i,n}} \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall n = 1,\ldots,|\mathcal{I}|, \\
& z^q_{i,n} \le \frac{q_{i,n} - r^q_{i,n}}{a^q_{i,n} - r^q_{i,n}} \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall n = 1,\ldots,|\mathcal{I}|, \\
& z^q_{i,n} \le \beta \frac{q_{i,n} - a^q_{i,n}}{a^q_{i,n} - r^q_{i,n}} + 1 \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall n = 1,\ldots,|\mathcal{I}|, \\
& z^c_{i,k} \le \gamma \frac{c_{i,k} - r^c_{i,k}}{a^c_{i,k} - r^c_{i,k}} \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall k = 1,\ldots,|\mathcal{K}|, \\
& z^c_{i,k} \le \frac{c_{i,k} - r^c_{i,k}}{a^c_{i,k} - r^c_{i,k}} \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall k = 1,\ldots,|\mathcal{K}|, \\
& z^c_{i,k} \le \beta \frac{c_{i,k} - a^c_{i,k}}{a^c_{i,k} - r^c_{i,k}} + 1 \qquad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall k = 1,\ldots,|\mathcal{K}|, \\
& v \le g_o \qquad \forall o = 1,\ldots,|\mathcal{P}_k| + |\mathcal{I}| + |\mathcal{K}|, \\
& g_o = o p_o - \sum_{s=1}^{|\mathcal{P}_k|+|\mathcal{I}|} l_{s,o} \qquad \forall o = 1,\ldots,|\mathcal{P}_k| + |\mathcal{I}| + |\mathcal{K}|, \\
& p_{o1} - l_{s,o1} \le z^q_{i,n} \qquad \forall s, o1 = 1,\ldots,|\mathcal{P}_k| + |\mathcal{I}| \quad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall n = 1,\ldots,|\mathcal{I}|, \\
& p_{o2} - l_{s,o2} \le z^c_{i,k} \qquad \forall s, o2 = 1,\ldots,|\mathcal{P}_k| + |\mathcal{K}| \quad \forall i = 1,\ldots,|\mathcal{P}_k| \quad \forall k = 1,\ldots,|\mathcal{K}|
\end{aligned}
\tag{B.3}
$$