Abstract

Colorings of geometric intersection graphs

Main inspiration of this paper are four intensively studied problems: the Hadwiger-Nelson problem, problems of online coloring of interval intersection graphs and disk intersection graphs, and Δ^2-conjecture for $L(2,1)$-labelings.

The Hadwiger-Nelson problem asks for the minimum number of colors required to color the Euclidean plane in such a way that any two points at distance one obtain distinct colors. Stated by Nelson in 1950, the problem inspired researchers in fields of geometry, combinatorics, topology and measure theory. Chapter 3 is devoted to a generalization of the problem where we color a graph $G_{[1,\sigma]}$ with $\mathbb{R}^2$ as a vertex set and edges between pairs of points at Euclidean distance in $[1,\sigma]$, for $\sigma \geq 1$. We consider b-fold version of the problem. Our method generalizes previous results and gives best known upper bounds on $\chi(G_{[1,\sigma]}$ for numerous values of σ. Using these colorings we introduce the notion of $L^*(2,1)$-labeling of the plane. It is a more restricted case of coloring and is used later in the thesis for online $L(2,1)$-labeling of disk graphs. We give a constructive method of finding b-fold $L^*(2,1)$-labelings of $G_{[1,\sigma]}$.

Fiala, Fishkin and Fomin [35] applied $L^*(2,1)$-labelings of $G_{[1,\sigma]}$ for online $L(2,1)$-labeling of σ-disk graphs, i.e. intersection graph of disks with diameters in $[1,\sigma]$. Inspired by their work in chapter 4 we present 4 algorithms designed for online σ-disk coloring using the b-fold version of colorings of the plane. We employ two techniques of vertex division before coloring - one is division according to the disk diameters, used before by Erlebach and Fiala [26], and the other is our own approach which is division between layers of the b-fold coloring of the plane. Some of the algorithms can be applied to online coloring other geometric shapes and online $L(2,1)$-labeling of disks and shapes. The algorithms yield better results in terms of their competitive ratios then the previously know methods.

In chapter 5 we study the problem of online coloring intersection graphs of intervals of bounded lengths. Interval graphs and their online colorings have been an object of research of Kierstead and Trotter [62]. They found an algorithm that uses at most $3\chi(G)-2$ colors and proved there is no better algorithms in general case. However, for unit intervals the best known algorithm is FirstFit, which uses at most $2\chi(G)-1$ colors and any algorithm can be forced to use at least $\frac{3}{2}\chi(G)$ colors. The gap between the results for unit and

general intervals inspired us to consider intersection graphs of intervals with bounded lengths, which are somehow similar to disks with bounded diameters. We show show that our method of coloring disks can be translated to bounded intervals, which gives us best known results for intervals of lengths in $[1, 2]$. We give lower bounds on the competitive ratio of any online coloring algorithm for bounded interval graphs, proving in particular that $\frac{5}{3}\chi(G)$ colors can be forced if we allow intervals of lengths in $[1, \sigma]$, for $\sigma > 1$.

The last chapter is about $L(2, 1)$-labeling of disks in an offline setting. The Δ^2-conjecture of Griggs and Yeh [44] asks if the $L(2, 1)$-span of any graph is bounded by $\Delta(G)^2$. We prove the conjecture for disk graphs with $\Delta(G) \geq 126$ by showing $\lambda(G) \leq \frac{4}{5}\Delta(G)^2 + 25\Delta(G) + 20$, as well as unit disk graphs with $\Delta(G) \geq 15$ for which we prove $\lambda(G) \leq 14\Delta + 11$.

Keywords: Hadwiger-Nelson problem, online coloring, disk graphs, interval graphs, $L(2, 1)$-labeling