Abstract

We prove that Sobolev-type embeddings of Slobodeckij spaces defined on a metric measure space are equivalent to the Ahlfors lower regularity of the underlying measure. Moreover, we show that Slobodeckij space defined on the Ahlfors lower regular metric-measure spaces can be continuously embedded into a fractional Hajłasz-Sobolev space, which is a special case of Hajłasz-Triebel–Lizorkin space. We also establish necessary and sufficient conditions guaranteeing compactness of embeddings of the Slobodeckij, fractional Hajłasz-Sobolev spaces, Hajłasz-Triebel–Lizorkin spaces, and Hajłasz-Besov spaces defined on the metric-measure spaces. Many results are illustrated by some examples. As a byproduct of our research, we obtain reinforced versions of analytical tools usefull in analysis such as characterization of totally bounded metric-measure spaces, improved Hanson's theorem and reversed Lusin's theorem.

Keywords: *metric-measure spaces, Sobolev spaces, fractional spaces, Ahlfors regularity, Rellich–Kondrachov theorem, Sobolev embeddings, compact embedding, Hajłasz-Triebel–Lizorkin spaces, Hajłasz-Besov spaces.*